# FIELDS AND WAVES

## 3.0 MAGNETIC FIELDS

The properties of "lodestone", a permanently magnetised mineral (magnetite ore), was certainly known to the Chinese as early as 290 - 300 BC. It behaves like a compass, pointing to the North, and so could be used in navigation. In 1269 Peter Peregrinus of Maricourt described experiments he had performed using very basic apparatus in a letter *On the magnet*. But it is William Gilbert who is usually referred to as the father of magnetism for his treatise *De Magnete* published in 1600, in which he set out the results of his experiments on magnets (and also on electrical phenomena). He was the first to propose that the Earth itself behaved like a large magnet, and that lodestones or pieces of iron magnetised by lodestones, were attracted to point towards the poles of the Earth in the way he demonstrated with a spherical "micro-earth", or globular lodestone.

A permanent magnet, such as a bar magnet or a compass needle - or indeed a piece of lodestone has two *poles*, one of which is attracted towards the North Pole of the Earth, so it is called the north pole of the magnet (although perhaps a better name would be the north- seeking pole of the magnet): the other pole is attracted to the South Pole of the Earth - it is the south pole of the magnet. It is quite easy to demonstrate with two magnets that like poles repel one another, whilst unlike poles attract. (One may conclude that, considering the Earth as a magnet, the magnetic pole near the geographic North Pole is in fact a south pole; and conversely the one near the geographic South Pole is a north pole!)

The magnetic north and south poles of the Earth are not coincident with the geographic poles; in fact they wander around, and over geological time there have even been episodes of reversal of the Earth's magnetic field. The mechanism that sustains the magnetization of the Earth is quite complicated, and includes electrical currents flowing in the molten iron core deep in the Earth's interior. A compass needle will indicate the direction of magnetic north; to derive from that the direction of true (geographic north) one needs to know the angle between these two directions, and this in turn is different from one place to another as well as changing a little from one year to the next. Maps will show this angle, the "angle of variation".

If the compass needle is allowed to turn not only in a horizontal plane, but also in a vertical plane, it will (except near the equator, where the magnetic field is essentially horizontal) point downwards (in the northern hemisphere), which is what you might expect on considering the Earth as a globular magnet. Here in London, the *dip angle*, the angle below the horizontal of the direction of the needle, is about 70o .

## 3.1 MAGNETIC FIELD LINES

You may be familiar with the pattern revealed by sprinkling iron filings on a sheet of paper placed over a bar magnet. This illustrates the *magnetic field lines* which are a useful aid to picturing a magnetic field. It is easy to experience the force exerted by magnet on another magnet, or by a magnet on a piece of iron or other material that responds to the magnetic field. We associate a magnetic field with the magnet in order to describe and understand these forces, and the iron filings help to visualize this field. The direction of the field at any point is the direction along which a tiny compass needle, free to rotate, will become oriented if placed at that point. And its strength will be related to the torque (twisting force) experienced by that compass needle if it is misaligned with the field.

So one way to plot out the field lines is to take a small compass and move it from place to place in the field, drawing a little arrow to show the direction taken up by the compass at each point. These then map out the field lines, and one finds that they appear to emerge from the north pole of a magnet and either go off "to infinity" or else end on a south pole. But please note that whenever there is a north pole, there is always a south pole. If you cut a bar magnet in two, you will not find one of the pieces with just a north pole and the other with just a south pole: you will find that you have now obtained *two* magnets, each with a north and a south pole.

A very important discovery was made by the Danish physicist Hans Christian Oersted whilst preparing demonstrations for a course of lectures he was going to deliver to students. As he reported to them, when he allowed an electric current to flow in a wire parallel to a nearby compass needle, the needle was deflected. By following the method described above for tracing out the pattern of field lines, one can show that they are arranged in concentric circles around the current-carrying wire. Their direction depends on the direction of the current in the wire. If you were to look along the wire in the direction of the current, the field lines are directed in a clockwise sense. (Put another way, if you were to drive a screw in the direction of the current, you would need to turn the screw in the direction of the field lines - assuming that is that the screw was the conventional right-handed screw!)

Suppose now that the current-carrying wire is wound as a *solenoid*; one finds that outside the solenoid, the field lines resemble those of a bar magnet. However, they continue *inside* the solenoid, and have neither a beginning nor an end. Indeed, inside a long solenoid, far enough from the ends that end effects can be ignored, the magnetic field is *uniform*, constant in magnitude and in direction, the direction being along the solenoid. So one end of the solenoid looks like the north pole end of a bar magnet, the other like a south pole. You can work out which end is which from the rule which determines the direction of the field lines round a current-carrying wire.

## 3.2 MAGNETIC FIELD STRENGTH

The picture we have so far described gives some indication of the magnetic field, but it is not yet sufficiently quantitative to be of practical use in deriving such interesting quantities as the force exerted by a magnetic field. The first step we need to take is to give a definition of the magnetic field strength. Since isolated magnetic poles - monopoles - do not, so far as we know, exist in Nature (poles always come in pairs, one north one south), it is not possible to follow the same path we used to define the gravitational field strength (the gravitational force exerted on a test mass divided by the mass of that test mass), or the electric field strength (the electrical force exerted on a test charge divided by the charge of that test charge). Nevertheless, historically, something like that was done by considering the force exerted on one of the poles of a *long* magnet, so long that the other pole could be considered to be far enough away as not to disturb the measurement. This approach is no longer used.

What is done is to exploit the fact that a magnetic field exerts a force on a wire carrying an electric current. Let us first consider the simplest case, where the wire is straight and perpendicular to the field to be measured. It is then found that there is a force on the wire that is proportional to the current carried by the wire, and is in a direction perpendicular both to the wire and to the field. The direction of this force may be specified by using the *left-hand rule.* If you arrange your *left* hand with the thumb, first finger and second finger each at right angles to the other two, and then point the **F**irst finger along the direction of the **F**ield and the se**C**ond finger along the direction of the **C**urrent in the wire, your thu**M**b will point in the direction in which the force on the wire will act to make it **M**ove. (It is not uncommon to see students in examinations on electromagnetism waving their hands about in strange ways!)

In a region where the field is uniform, the force on the wire is proportional to the length $L$ of the wire in the field; and, as said, it is also proportional to the current $I$ carried by the wire. The force on the wire may then be used as a measure of the strength $B$ of the magnetic field in this region:

$$B = \frac{F}{IL}.$$

The unit for magnetic field strength is tesla (T): T = N.A$^{-1}$ .m$^{-1}$.

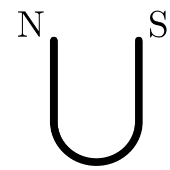
More generally, if the wire and the field are not at right angles to one another, the formula becomes:

$$F = BIL\sin\theta,$$

where $\theta$ is the angle between the direction of the field and the direction of the current in the wire. (The force is zero if the field and the current are parallel). The direction of the force is still at right angles to both the field and the current, and in the direction still given by the left-hand rule, but with the first finger pointing in the direction of the component of the field **B** perpendicular to the wire.

## 3.3 TORQUE ON A COIL

If a coil of wire carrying an electric current is placed in a magnetic field, the resulting forces acting on it are in general such as to have a turning effect on the coil - they comprise a *couple* or *torque.* Let us start by considering a simple example. Suppose we have a large magnet shaped like this:



so that the poles are opposite to one another. Between the poles we may assume that the magnetic field $B$ is uniform, and if the magnet held in a horizontal plane, the field direction will also be horizontal. Suppose now that we make a single rectangular loop of wire arranged so that two opposite edges, each of length $l$ are vertical, and the other two, each of length $d$ are horizontal, and place it between the poles of the magnet. We suppose it to be small enough that the magnetic field running through it is everywhere the uniform horizontal $B$-field. Consider first the case when the rectangular loop of wire is in a vertical plane parallel to the direction of the field, and that there is a current $I$ around the loop. Each of the horizontal sides of the rectangular loop is then parallel to the direction of the field, so although each carries a current, there is no force on it. However, the current $I$ in one of the vertical edges of the loop will be running upwards, whilst in the opposite side it will be running downwards. In either case, it will be perpendicular to the field, and so there will be a force of magnitude $F = BIl$ on this segment length $l$ of the wire, in each case perpendicular both to the wire and to the field. But the forces on these two edges of the loop, although having the same *magnitude*, will be in *opposite* directions (since the direction of the current flow is opposite along these two edges). So the net force on the loop of wire is zero.

However, forces of equal magnitude but opposite directions acting on different parts of the loop (as these do), will exert a turning effect on the loop - a torque or couple. The magnitude of this couple $C$ is the product of the force $F$ with the distance between the lines of action of the forces; in this case this distance is just $d$, the length of the horizontal edges of the rectangle:

$$C = Fd = BIld = BIA,$$

where $A$ is the area ( $= ld$ ) of the rectangular loop. If now instead of a single loop of wire, we had a coil made from $N$ identical loops, the result would be

$$C = NBIA.$$

And if the coil with $N$ windings had been in a plane which, although still vertical, had made an angle $\theta$ with the direction of the magnetic field $\mathbf{B}$ the magnitude of the couple would be

$$C = NBIA\cos\theta;$$

(cosine, not sine, because the angle between the direction of the force on the vertical section of wire and the plane of the coil is $90^\circ - \theta$). In each case, the direction of the couple is such as to turn the coil to lie in a plane perpendicular to the field lines, when the couple vanishes.

## 3.4 THE MOVING COIL METER

We can exploit the torque exerted on a coil carrying a current $I$ to *measure* the current (assuming that we already know the field strength $B$). This is done by balancing the torque against another mechanically produced torque that can be measured. The first step is to form the pole faces of the magnet in such a way that the space between them is cylindrical, and to place a cylindrical piece of "soft" iron in this space leaving a narrow gap between the iron core and the pole faces. The effect of this is to make the field in this gap *radial.* A coil such as we have been considering is then suspended around the soft iron core, with its windings in the gap between the core and the poles of the magnet. It is free to rotate about an axis coincident with that of the cylindrical core and the shaped pole faces.

With this arrangement, the field is always perpendicular to the vertical edges of the coil, and the torque on the coil is given by $C = NBIA$, so long as the coil remains in the field. Unless something is done to stop it, the coil will be turned until it is no longer in the field. But something *is* done, because as part of the suspension, there are attached hairsprings which resist the turning of the coil. The more the coil is turned, the greater is the torque exerted by the hairsprings opposing the turning of the coil, until at equilibrium, the torque exerted by the hairsprings exactly balances that produced by the current in the coil.

But the torque exerted by the hairsprings is (to excellent approximation) proportional to the angle $\theta$ through which the coil has been turned. So now all that is needed is to measure this angle $\theta$, and to determine the factor of proportionality relating the couple $C$ to $\theta$ in the equation $C = k\theta$. This can be done in a separate experiment. We then have

$$\theta = I \cdot NBA/k,$$

and if there is a scale on which a pointer moves to indicate the angle $\theta$, it can in fact be calibrated to show instead the value of the current $I$, since they are proportional to one another, and the factor of proportionality is known.

## 3.5 MOTORS

The torque on a coil carrying a current in a magnetic field can be exploited to construct a motor. The main problem to be overcome is that he torque acts in such a way as to turn the coil until it is in a plane perpendicular to the field, but it then vanishes. This problem can be overcome in a number of different ways. Possibly the simplest method is to arrange the way that the current is fed to the coil from the battery or other (d.c.) source (which in any case has to be done so as to enable the coil to rotate indefinitely without twisting up the wires!) reverses the direction of the current in the coil every time it has turned through 180° This can be done with a *split-ring commutator*. (See the diagrams in the text-book for a clear illustration and discussion. There is also a more complete treatment of different kinds of electric motor).

## 3.6 CHARGED PARTICLES IN MAGNETIC FIELDS

An electric current is, after all, a consequence of electric charge in motion. So you might not be surprised to learn that a magnetic field exerts a force on a charged particle moving through it. When the velocity $\mathbf{v}$ of the charge is perpendicular to the magnetic field $\mathbf{B}$, the force $\mathbf{F}$ is perpendicular to both of them, and for a charge q its magnitude is given by

$$F = qvB.$$

More generally, if the angle between $\mathbf{v}$ and $\mathbf{B}$ is $\theta$, the force has a magnitude

$$F = qvB\sin\theta$$

and (for positive charge $q$) it is in the direction consistent with the left- hand rule, where now the second finger points along the direction of the velocity. [For those who enjoy using vectors, this result can be expressed through the equation

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B},$$

where the expression $\mathbf{v}\times\mathbf{B}$ is a vector with magnitude $vB\sin\theta$ in the direction as specified by the left-hand rule.] This force is responsible for the deflection of the tracks of elementary particles (protons, electrons and the like) which you will probably have seen in photographs of these tracks in particle detectors which usually have large magnetic fields for just this purpose: for from the deflection it is possible to determine the component of the momentum of the particle at right angles to the magnetic field. In a like fashion, the same force which deflects the motion of charged particles through the magnetic field, is exploited in many different devices which use beams of charged particles, be they particle accelerators, storage rings, electron microscopes, controlled nuclear fusion devices, ...

Another consequence of this force is the so-called *Hall effect*. When a current flows in a conductor or a semiconductor, charge is transported against the resistance of its flow by the application of the voltage which drives it. This flow of current is best envisaged as a *drift* of the charged carriers (which can be negatively charged electrons, or as is true for some kinds of semiconductors, *positively* charged "holes"). If a strip of the conductor or semiconductor carrying a current is placed in a magnetic field perpendicular to the direction of the current, these carriers will experience a force; and if the arrangement has, for example, the field vertically downwards, and the conducting strip is broad but thin, and lies horizontal, magnetic force will deflect the carriers towards the thin, vertical, edges of the strip. This will result in a build-up of charge on one of these edges, and a corresponding decrease of charge on the opposite edge. There then results in a voltage difference *across* the strip, produced by these opposite charges - the *Hall potential difference*, $V_h$ . Eventually, this build-up of charge is sufficiently large that the voltage difference it produces acts on the moving carriers of the current (through the *electric* force it produces - which is always in the opposite direction to the magnetic force which pushed the carriers to the edge of the strip in the first place) in such a way as to balance exactly the magnetic force. Now the electric *field* strength $E$ associated with the build-up of carriers is

$$E = V_h/d$$

and this exerts a force $F$ on a carrier with charge $q$ given by

$$F = qE = qV_h/d.$$

So when this is in balance with the magnetic force we have also

$$F = qvB,$$

so that

$$V_h = vBd.$$

But now we need to determine the speed $v$ of drift of the carriers along the conducting strip. The current $I$ along the strip is proportional to $v$; in fact if the number of carriers in each $\mathrm{m}^3$ of the strip is $n$, and the cross-section area of the strip is $A$, it follows that the current $I$ carried along the strip is

$$I = nAvq.$$

But we also have $A = dx$, where $x$ is the width of the strip. Thus

$$V_h = BI/nxq.$$

And the Hall voltage $V_h$ can be measured. Its *sign* will tell the sign of the charge carriers, and its magnitude can be used to determine their density $n$, since one may assume that the magnitude of $q$ will usually be $e$, that of the charge on an electron. Alternatively, the Hall effect can be exploited to measure the strength $B$ of a magnetic field with what is called a *Hall probe.*

## 3.7 THE FIELD IN A SOLENOID

The field in a solenoid carrying a current can be measured, or example, using a Hall probe. It is found that the field strength is proportional to the current $I$ carried by the solenoid, and also proportional to the number of windings per metre along its length. Inside the solenoid (apart from end effects) it is uniform, and parallel to the length of the solenoid. If the total number of turns of the solenoid is $N$, and its length is $l$, we then have for the field strength $B$ inside the solenoid

$$B = \mu_0 \frac{IN}{l},$$

where $\mu_0$ is a fundamental constant called the magnetic permeability of free space. It is *defined* to have the value

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H \cdot m^{-1}},$$

where H stands for henry, named for the American physicist Joseph Henry. It is the unit for *inductance* which we will meet later.

### 3.8 THE FIELD AROUND A LONG STRAIGHT WIRE

We saw earlier that the field lines around a long straight wire are concentric circles centred on the wire, and in planes perpendicular to it. What we now have to consider is how for a given current $I$ in the wire, the strength $B$ of the field varies with the radius of these circles, that is to say with the distance $r$ from the wire (symmetry arguments can be used to show that there is nothing else on which it can depend). Experiment shows that $B$ is proportional to $I$, and also that it varies like $1/r$. It was Ampère who first derived the factor of proportionality which should appear in this formula. He first considered a long solenoid, for which the formula giving the field strength inside the solenoid has been given (in modern terms) above. He then argued that the same formula would still apply even if the solenoid were bent around into a toroidal solenoid - like a bicycle tyre. So we would still have

$$Bl = \mu_0 NI.$$

If we were to consider any line of the magnetic field inside the solenoid, along which the field strength would be $B$, this formula would apply, but now $l$ could be interpreted as the distance around the field line, and $NI$ as the total current passing through the loop which constituted that field line. He then generalised that idea to the case of the long straight wire, where the loop was the circle of radius $r$ which has a length $2\pi r$. And the current passing through that loop is just the current $I$ carried by the wire. The result is then

$$B = \frac{\mu_0 I}{2\pi r}.$$

This is an example of a more general rule named after Ampère.

### 3.9 THE DEFINITION OF THE AMP

At last we are in a position to give the definition of the amp, from which will follow that of the coulomb, the volt, etc. The fact that there is a magnetic field around a wire carrying a current, and that a magnetic field exerts a force on a wire carrying a current, allows us to relate the magnitude of the current carried in each wire to a force which we can measure. And that allows us to define a standard for the current, the amp. If we have two parallel long straight wires, distance $d$ apart, with a current $I_x$ in one wire (call it wire $X$), and likewise a current $I_y$ in the other wire we shall call wire $Y$, then the field strength at wire $Y$ produced by the current in wire $X$ is

$$B = \frac{\mu_0 I_x}{2\pi d}$$

and this exerts a force on each length $L$ of the wire $Y$ given by

$$F = BI_yL = \frac{\mu_0 I_x I_y L}{2\pi d}.$$

The force is attractive if the current flows in the same direction in each of the wires.

So we may define the amp (A) as follows: If in each of two infinitely long parallel wires 1 m apart in vacuum the same current flows, and the force on each of the wires is $2 \times 10^{-7}$ N $\cdot$ m$^{-1}$, the current in each of the wires is 1 A.

## 3.10 MAGNETIC MATERIALS

Some materials, for example iron, can be "permanently" magnetised: they are called ferromagnets. The explanation for their magnetisation involves an understanding of the magnetic properties of individual atoms, and indeed of the electrons and atomic nuclei from which they are built. The elementary particles like the electron and the proton themselves behave like tiny magnets, and it was a great triumph of quantum mechanics to explain why this should be so. In addition, the motion of the electrons around the atomic nucleus can be thought of as a tiny electric current loop, with which there is associated a magnetic effect. So atoms can behave like tiny magnets. In a ferromagnet, it is energetically favoured for these tiny magnets to become aligned, all pointing in the same direction in a piece of the material; such a piece of a ferromagnet in which the atomic magnets are all aligned is called a domain. But the domains, although large compared to atoms, can be quite small; and the alignment can be different from one domain to the next. So in an unmagnetised piece of iron, the magnetisation of the domains is pretty well random in direction, and the overall magnetisation vanishes.

However it is relatively easy to make the magnetisation of each of the domains line up with an externally imposed magnetic field, and what special about permanent magnets is that once aligned they stay aligned until something happens to make them point in a different direction. Ferromagnetic materials differ one from another in the difficulty with which they can be magnetised, or the direction of their magnetisation can be changed. If it is easy, the material is said to be a “soft” magnetic material (iron for example); if it is more difficult, the material is said to be a “hard” magnetic material (most kinds of steel, for example).

When a ferromagnetic material is placed inside a solenoid carrying a current, the magnetic field produced by the current in the solenoid can magnetise the ferromagnet, and so it too will produce a magnetic field which enhances that of the solenoid alone. This means that our formula for the magnetic field strength in a long solenoid (or a toroidal solenoid) has to be modified. Instead of $B = \mu_0 n I$ ($n$ being the number of turns per metre along the solenoid), we will now have

$$B = \mu_r \mu_0 n I,$$

where $\mu_r$ is a constant characteristic of the material inserted into the solenoid called its relative permeability ; this can have a value as high as 2000 for soft iron. You should exercise caution when using this formula, since it only holds in a regime in which the fields are not too large; the field should not exceed the saturation field strength, at which the magnetisation of the material can be increased no further.

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