FIELDS AND WAVES

2.1 ELECTRICAL FIELDS

You are no doubt familiar with the fact that many materials (for example perspex) can be electrically charged by rubbing them. The ancient Greeks had observed that amber when rubbed with fur was able to attract light objects: the Greek word for amber is electron, and it is from this that we derive our word electricity. It was Benjamin Franklin (1706-1790), the American statesman, diplomat and scientist, who first recognised that what had previously been considered as two different kinds of electricity were in fact related: negative electric charge was simply obtained by removing positive charge from an electrically neutral body. (Unfortunately his choice of a sign convention was "wrong", and the charge on the electron is negative, so that a negatively charged body has an excess of electrons, and a positively charged body has a deficiency of electrons!)

Some substances become positively charged by friction, others become negatively charged. It was found quite early in the course of scientific investigations of electrical phenomena that like charges repel, and unlike charges attract. In an electrically neutral object (one which carries neither positive nor negative electrical charge overall), there are the same number of electrons as protons. Electrons carry a negative fundamental unit of charge, usually written as $-e$, and protons an equal amount of positive charge $+e$, so the net result is to give an overall zero amount of charge (since electric charge can be added algebraically). In some materials, the electrons can move about quite freely, and they are said to be good conductors of electricity; most metals are excellent conductors. Other materials on the contrary have essentially no free electrons, and they are insulators ; glass or perspex are examples of insulators. Still other materials stand somewhere between these two extremes, and they include the technologically important semiconductors. An electric current is produced whenever electric charge moves from one place to another. In its simplest form this might be the movement of electrons along a conducting wire. The basic standard from which electrical units are derived is the unit of electrical current, the amp, named for the physicist Ampère (1775-1836). The abbreviation for this unit (which we will define later) is A. The unit of charge is the coulomb (C), named for the physicist Coulomb (1736- 1806). This is the amount of charge carried each second by a steady current of 1A. The charge on an electron is $-e$, where

$$e = 1.6 \times 10^{-19} \text{ C}.$$

The fact that electrically charged bodies exert forces on one another suggests that we introduce the concept of an electric field associated with electric charge. We say that the source of the electric field is electric charge in much the same way that we might say that the source of the gravitational field is mass.

2.2 ELECTRIC FIELD STRENGTH

We found it convenient to define the gravitational field by the force it exerted on a test-mass. Since electric charge is the source of the electric field, it follows from Newton's third law that it is electric charge on which the electric field exerts a force. So it is convenient to define the electric field strength $\mathbf{E}$ to be the electric force exerted by the field on a test-charge q divided by q.

$$\mathbf{E} = \mathbf{F}/q.$$

Since the unit of force is N and that of charge is C, the unit of $\mathbf{E}$ will be N.C^{-1}. A uniform field exists in a region in which $\mathbf{E}$ is constant, both in magnitude and direction. A useful way to picture an electric field is by drawing field lines which show at each point the direction of the field, and can also be used to indicate the strength of the field by having them bunched together where the field is strong and spread apart where it is weak. The field lines start on positive charges and end on negative charges (which for that reason are sometimes called the sources and sinks respectively for the field).

2.3 THE ELECTRIC POTENTIAL

To move a test charge q from one point to another in an electric field requires that work should be done (a positive amount of work if q is positive and is moved against the direction of the field and so on). This means that we can associate a potential energy to the test charge at a point P in the field, being the amount of work needed to move the charge from P to infinity. The electric potential, V, at the point P is then defined to be the electric potential energy of the test charge q, divided by q.

$$V(P) = (\text{PE of } q \text{ at } P)/q.$$

The unit of electric potential is the volt (named for the physicist Volta (1745-1827)), abbreviated V. You should be able to see that the unit for **E**, which we had previously given as N.C^{-1} could just as well be (and more often is) written as V.m^{-1}. Just as in the case of the gravitational field, we can define equipotential surfaces, and again the field lines are orthogonal (perpendicular) to the equipotential surfaces, and the field is minus the gradient of the potential.

$$\Delta V = -\mathbf{E} \cdot \Delta r.$$

2.4 CAPACITORS

A capacitor is a device for storing electric charge. The simplest example is a pair of parallel metal plates separated from one another and supported by insulators. If the charge stored on one of the plates is Q and the potential difference (voltage difference) between them is V, experiment shows that Q is proportional to V. It is also proportional to the area A of the plates, and inversely proportional to the distance d separating them. If the gap between the plates is empty, the factor of proportionality is written as ϵ_0, a fundamental constant called the permittivity of free space. So one has the formula

$$Q = \frac{AV\epsilon_0}{d}.$$

The electric field (apart from "edge effects " near to the edges of the plates) is uniform in the region between the plates, and is in the direction from the plate with the higher potential towards the one with the lower potential, perpendicular to the plates. Remembering that the field is minus the gradient of the potential, it follows that the magnitude of the electric field between the plates is given by

$$E = V/d = \frac{Q}{A\epsilon_0}.$$

The unit for ϵ_0 is F/m , where F stands for farad (named for Faraday (1791-1867)), and is the unit of capacitance. 1F = 1 C/V. The accepted value of ϵ_0 is

$$\epsilon_0 = 8.854\,817\,817\ldots \times 10^{-12}\ \mathrm{F/m}.$$

If the space between the plates of a parallel plate capacitor is filled with a dielectric, the capacitance changes. A dielectric is an insulating material chosen so as to increase the capacitance. It does this because the individual molecules in the dielectric become electrically polarised, which is to say that within the molecule the electrons (which are negatively charged) are pulled a little towards the positive plate of the capacitor, which results in a net negative charge accumulating on the face of the dielectric nearest the positive plate of the capacitor, and likewise a net positive charge left behind on the surface of the dielectric closest to the negatively charged plate of the capacitor. The negative surface charge near to the positive plate of the capacitor can then push electrons off from that plate towards the battery (or whatever else it was which had charged the capacitor), so making the charge on the plate still more positive: and conversely the positive charge on the surface of the dielectric nearest the negative plate of the capacitor attracts still more negatively charged electrons to that plate, making its charge still more negative. What all this means is that the amount of charge stored on each plate of the capacitor has been increased, and so its capacitance is greater. The ratio of the capacitance with a dielectric filling the space between the plates to that without the dielectric (which is the same at a fixed voltage difference between the plates as the ratio between the charge stored with the dielectric to that without) is called the relative permittivity of the dielectric (or the dielectric constant) of the dielectric: it is written ϵ_r. So the charge stored on the capacitor is now

$$Q = \epsilon_r Q_0 = \frac{A\epsilon_0\epsilon_r V}{d},$$

where Q_0 is the charge stored without the dielectric in the gap between the plates. The capacitance C of a capacitor is the amount of charge stored per unit of potential difference across its plates. Thus for a parallel plate capacitor,

$$C = Q/V = \frac{A\epsilon_0\epsilon_r}{d}.$$

The plates of a capacitor need not be as in the example we have been discussing. One "plate" could be your body, and the other the Earth! This would only make a practical capacitor if you were electrically insulated from the Earth, for example by standing on a rubber mat. You could then be made to store an electric charge by being connected to a device that generated a potential difference between your body and the Earth. The small capacitors you may have seen inside electronic devices, or radio or TV sets, are made by wrapping two strips of aluminium foil (which act as the plates), into a tight cylinder: there is a dielectric insulation between the strips of foil. The capacitance of such a capacitor is usually of the order of a few microfarad (μF). In other capacitors used in tuned circuits such as in radio receivers, the capacitance can be varied by altering the area of overlap between the plates, so changing the effective area of the capacitor.

2.5 ENERGY STORED IN A CHARGED CAPACITOR

Work has to be done to increase the charge on a capacitor from Q to $Q+\Delta Q$. The work needed to be done to charge it from zero charge to a final charge Q_f can be determined by adding together the work needed to go from zero to Q_f in a succession of small steps. And the final result is then the energy stored in the capacitor, since the work done can be recovered as the capacitor discharges, for example through a resistor where the current produced by the flow of charge would generate heat.

The work done in changing the charge from Q to $Q + \Delta Q$ is $V\Delta Q$. So the energy stored, which is the work done in going from zero charge to charge Q_f is obtained by adding together all the incremental amounts $V\Delta Q$, and this in turn (in the limit where ΔQ is made arbitrarily small) is just the area under the graph of V against Q. So the energy stored is

$$E = Q_f V_f/2 = CV_f^2/2.$$

2.6 COULOMB'S LAW

The French physicist Charles Coulomb, using a torsion balance, established a law for the force between electric charges which is reminiscent of that proposed by Newton for the gravitational force between masses. It is that the force is proportional to the product of the charges and inversely proportional to the square of their separation, so for point charges Q_1 and Q_2 distance r apart we have

$$F = k\frac{Q_1 Q_2}{r^2},$$

where k is a constant which can be shown to be equal to $1/4\pi\epsilon_0$. Thus we have

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}.$$

Note that it is a repulsive force if the two charges have the same sign, and is attractive if they have opposite signs.

The force on a test charge q placed at distance r from a charge Q (which is either a point charge, or one small enough in dimension to be regarded as a point; or as it turns out, any spherically symmetric distribution of charge) is then $F = \frac{qQ}{4\pi\epsilon_0 r^2}$, and since the field strength E is defined by F/q, it is given by

$$E = \frac{Q}{4\pi\epsilon_0 r^2}.$$

The electric potential produced by the charge Q may be deduced following arguments, essentially the same as those which allowed us to determine the gravitational potential from the gravitational field around a mass M. The result is

$$V_E = \frac{Q}{4\pi\epsilon_0 r}.$$

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