THE AETHER ISSUE

-The function of a travelling SHM that propagates on positive direction of Ox axis is:

$$y(x,t) = A\sin(kx - \omega t + \varphi) \qquad (1)$$

When deriving a function of several variables, one has to deal with *partial derivatives*. We will use the function (1) to find the relation between its partial derivatives;

$$\frac{\partial y}{\partial x} = k * A\cos(kx - \omega t + \varphi)\,; \quad \frac{\partial^2 y}{\partial x^2} = -k^2 * A\sin(kx - \omega t + \varphi)\,; \quad \text{So, } \frac{\partial^2 y}{\partial x^2} = -k^2 y \qquad (2)$$

$$\frac{\partial y}{\partial t} = -\omega * A\cos(kx - \omega t + \varphi)\,; \quad \frac{\partial^2 y}{\partial t^2} = -\omega^2 * A\sin(kx - \omega t + \varphi)\,; \quad \text{So, } \frac{\partial^2 y}{\partial t^2} = -\omega^2 y \qquad (3)$$

From the equation (3) we isolate $y = -\frac{1}{\omega^2}\frac{\partial^2 y}{\partial t^2}$ and by substituting in (2) we arrive at the wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 y}{\partial t^2}$ as $\frac{k}{\omega} = \frac{1}{v}$ we get

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad (4)$$

-The equation (4) relates the "**displacement**" changes in space domain with respective **displacement** changes in time domain. It is **<u>valid for all types of waves</u>** no matter what kind oscillation of it is. The parameter "***v***" stands for the ***propagation speed*** of the considered "displacement" in a physical medium. Remember that the propagation *<u>speed depends on medium characteristics</u>* . (Example; Travelling SHM along a string $v = (T/\mu)^{1/2}$).

-By the end of 19^{th} century, the scientists proved the ***wave nature of light*** (Young exp) and found that it is a ***transverse wave***. Like all other waves, the *light wave* has to obey to the equation (4). Maxwell showed that wave lights are part of a bigger family; the *electromagnetic waves*. He showed that for all the electromagnetic waves, the ***electric field*** stands for "***wave displacement***". He named ***c*** their propagation ***speed in vacuum***. Maxwell showed that the wave equation for light waves

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \qquad (5)$$

is valid for all electromagnetic waves. He even expressed the speed "***c" as*** a function of parameters of a propagating media. But, what is the medium that propagates the light in "empty space" between the stars?. He proposed the presence of a ***special medium that fills the whole universe*** and supports the ***propagation*** of light and other ***electromagnetic waves***. The physicists called it "***aether"***.

-The physicist community welcomed very pleased the aether because it would be "*the absolute frame for the physics' laws*". All other reference frames would be at rest or in absolute motion with respect to this medium. The "displacements" of this medium would correspond to electric oscillations and it would propagate electromagnetic waves. But, the aether had to: **a)** penetrate all matter object; **b)** be *weightless;* **c)** be *extremely elastic* and other strange characteristics.
Anyway, to accept its existence the physics has to prove it experimentally.
*H**ow to prove experimentally the "aether" existence***!

THE EXPERIMENT MICHELSON-MORLEY

-This experiment remained in the science history as a major case that demonstrates that in physics, no theoretical development is accepted without experimental proves. To verify the aether existence, Albert Michelson and Edward Morley had this idea; **as the earth moves (proved experimentally) around its axis and around the sun, for sure it is in movement toward the aether**. To prove this movement, they thought to use the Michelson interferometer as the most precise device of the time.

- At start, they aligned the arm PM1 (fig. 1) along the direction of earth movement around the sun (v = 30Km/s). The path difference between the two interfering wavelets is completed at beam splitter P, in their way back.

For the arm PM1; When the wavelet arrives at point M1 (*time* t_{for}), this point (fig.1.a) is advanced by $v*t_{for}$ with respect to its initial position on aether frame. So, its first path is:

$$c*t_{for} = L_0 + vt_{for} \rightarrow t_{for}(c-v) = L_0 \_and\_ t_{for} = L_0/(c-v) \qquad (6)$$

The wavelet path in aether is shorter ($L_0 - v*t_{back}$) when returning from M_1 to P**.** So, we get

$$c*t_{back} = L_0 - vt_{back} \rightarrow t_{back}(c+v) = L_0 \_and\_ t_{back} = L_0/(c+v) \qquad (7)$$

The total time spent by this wavelet along the arm PM1 is

$$t_1 = t_{for} + t_{back} = \frac{L_0}{c-v} + \frac{L_0}{c+v} = \frac{L_0(c+v) + L_0(c-v)}{c^2 - v^2} = \frac{2L_0 c}{c^2(1 - v^2/c^2)} = \frac{2*L_0/c}{1 - v^2/c^2} \qquad (8)$$

For the arm PM2; When the wavelet arrives at point M2 (*after time* t_{for}), this point is advanced by $v*t_{for}$ on the right. So, the real forward path would be (see fig 1.b)

$$(c*t_{for})^2 = L_0^2 + (vt_{for})^2 \rightarrow t_{for}^2(c^2 - v^2) = L_0^2 \_and\_ t_{for} = \frac{L_0}{\sqrt{c^2 - v^2}} = \frac{L_0/c}{\sqrt{1 - v^2/c^2}} \qquad (9)$$

When returning from M_2 to P, the wavelet travels the same path length (fig.1.b)**.** So, we get

$$t_2 = t_{for} + t_{back} = \frac{2*L_0/c}{\sqrt{1 - v^2/c^2}} = \frac{2*L_0/c}{(1 - v^2/c^2)^{1/2}} \qquad (10)$$

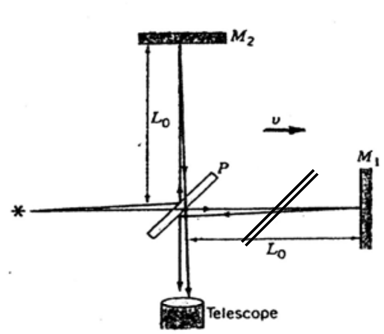
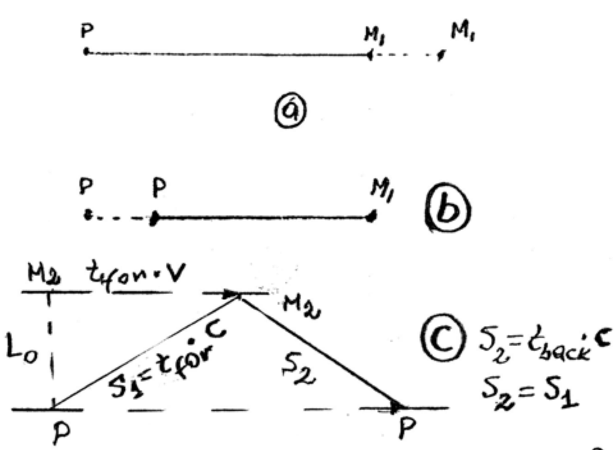


Fig.1



- So, when meeting "anew" at bean splitter P, the wavelets have a ***time difference***

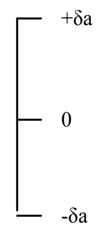
$$\Delta t_a = t_1 - t_2 = \frac{2*L_0/c}{1-v^2/c^2} - \frac{2*L_0/c}{(1-v^2/c^2)^{1/2}} = 2*L_0/c\left[\frac{1-(1-v^2/c^2)^{1/2}}{1-v^2/c^2}\right] =$$
$$2*L_0/c\left[\frac{1-1+0.5*v^2/c^2}{1-v^2/c^2}\right] = 2*L_0/c\left[\frac{0.5*v^2/c^2}{1-v^2/c^2}\right] = \frac{L_0}{c}\left(\frac{v}{c}\right)^2 \qquad (11)$$

Note the two approximations; $(1-x)^k \sim 1 - k*x$ and $1-(v/c)^2 \sim 1$.

We find that the ***path difference*** between the two wavelets is $\delta_a = c*\Delta t_a = L_0\frac{v^2}{c^2}$ (12)

And the phase shift $\Delta\varphi = \mathbf{2\pi/\lambda*\delta_a}$ will produce a bright or dark fringe at the telescope crosshair. Then, if one rotates 90^0 the interferometer arms, the PM_2 substitute PM1 and one finds a new ***time difference***

$$\Delta t_b = t'_1 - t'_2 = t_2 - t_1 = -(t_1 - t_2) = -\frac{L_0}{c}\left(\frac{v}{c}\right)^2 \qquad (13)$$



and the corresponding new ***path difference*** $\delta_b = c*\Delta t_b = -L_0\frac{v^2}{c^2} = -\delta_a$ (14)

So, during the rotation, the ***path difference*** between the two interfering wavelets **changes by**

$$\delta = 2\delta_a = 2*L_0\frac{v^2}{c^2} \qquad (15) \qquad \delta = 2*10m\frac{(30x10^3\,m/s)^2}{(3x10^8\,m/s)^2} = 200x10^{-9}\,m = 200nm$$

Based on this calculation, Michelson and Morley waited to see an easily observable shift of fringes in the telescope crosshair but they could not observe anything that could be considered as a real shift. **All measurements gave 0-shift inside the experiment uncertainty**.

- For a long time different scientist repeated the experiment but the result was the same. The negative result needed an explanation. Two main explanations remained:

a) There is no aether, so the upper scheme of calculations is wrong.
b) The arm of interferometer (PM_1) aligned with movement direction **contracts** and this contraction produces 0-path difference between the two arms. In this case we have

$$t_1 - t_2 = \frac{2*L_1/c}{1-v^2/c^2} - \frac{2*L_0/c}{(1-v^2/c^2)^{1/2}} = 0 \rightarrow\rightarrow L_1 = L_0(1-v^2/c^2)^{1/2} \qquad (16)$$

COVARIANCE

- The concept of aether entered in physics through the theory of electromagnetism developed by Maxwell. This theory was the major achievement in physics from the time of Newton. It is a very compact system of equations that permits to derive explications for all E.M. phenomena. All scientific community appreciated it very much and this explains why for a long time the scientists tried to prove the presence of "aether" .

-But the theory of Maxwell had another week point**; it was not covariant**. Meanwhile the mechanics' laws are **covariant**; **that is they keep the same form with respect to Galilean transformations**. For example, if we study the movement of an object with masse ***m*** in an ***inertial frame***, we know that the Newton's laws are valid and we can write the second law of Newton

$$F = m * a \quad (17)$$

The fact that mechanics' laws are ***covariant means that if we study the movement of the same object into another frame that moves with constant speed with respect to the first frame, the second law of Newton does not change form***

$$F_1 = m * a_1 \quad (18)$$

We know that the Galilean transformations tie the coordinates and time in the two systems as follows

$$x_1 = x - v * t; \_and \_ t_1 = t; \quad (19)$$

So, as $$a_1 = d^2 x_1 / dt_1^{\,2} = d^2 x_1 / dt^2 = dv / dt = a \quad (20)$$

The mass of object does not depend on the frame. We find the same value of the force in the new frame, $F_1 = m * a = F$ and this proves the covariance of second law. Without entering in details, we note that all mechanic laws and conservation principles (energy or momentum conservation...) are covariant with respect to Galilean transformations. This is a very important quality of mechanics' laws because they do not have special requirement on the frame selection: they do apply and are expressed equally to all "***inertial frames***".

- As mentioned the theory of Maxwell is not **covariant**. Here are two examples;

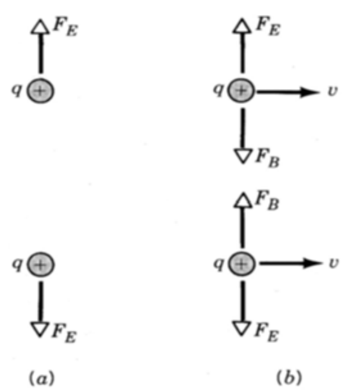


Fig 2

a) Two equal positive charges moving with same velocity. If studied with respect to a frame that moves with the same velocity, the charges are at rest (fig2.a). As consequence, they push each other with the electrostatic force F_{el}. If studied with respect to the laboratory frame, each moving charge creates a magnetic field (i.e.force F_B) that attracts the other charge. In this frame each (fig 2b) charge is submitted to the resultant force $\vec{F} = \vec{F}_E + \vec{F}_B$ So, in two different frames related by Galilean transformations there are different result. In this frame each charge is submitted to the resultant force $F = F_{el} - F_B$. So, in two different frames related by Galilean transformations there are different result.

b) When the Galilean transformations are applied to the wave equation (5) its form changes.

- The missing of covariance for Maxwell equations was a big handicap because one had to say that electromagnetic theory was right only in one specific frame. Meanwhile its validity was proved by explaining numerous experiments. The other possibility was to accept that the Galilean transformations were not right. But their validity was proved in mechanics.

- Einstein thought was the following: Or the Maxwell theory was to be *corrected* or the Galilean transformations were to be corrected. He decided ***to revise the Galilean transformations and the mechanics laws and correct them so that the motion of objects with very high speed (like light particles = photons) be described in conformity to experimental results.*** To make these corrections he introduced the two postulates that are the basics of the theory of special relativity.

THE TWO POSTULATES

- This theory is known as the **special** theory of relativity because its **two** basic **postulates,** *like mechanic's laws***,** apply **only for inertial frames(as for Newton's laws)**. The first principle is both a restriction and a guide for the formulation of any physical theory. It requires "***covariance" for all physical laws***. The principle of relativity: ***All physical laws have the same form in all inertial frames***. Since all inertial frames must offer the same law formulation, there is *no need for absolute frame "aaether-tied"*. Maxwell equations have to be covariant like all other physics' laws. Einstein ***corrected the Galilean transformations*** so that the E.M. theory of Maxwell is covariant, too.

- The second principle deals with the constancy of light speed: ***The speed of light in vacuum is the same in all inertial frames of reference and it is independent on the relative motion of the source or the observer.*** If we refer to our everyday experiences, it is difficult to accept this postulate. In fact, even Einstein did not pretend to explain why it is like that. He just formulated the result of multiple experiments. So, we have to accept this principle with two justifications: *our experience does not cover so high values of speed and the experiment results show that this is true*.

IMPORTANT CONSEQUENCE OF SECOND PRINCIPLE: ***An inertial observer (or any material particle) cannot travel at speed c. The speed of light in vacuum is a maximum unattainable by any object.***
Let's prove it by showing that traveling with speed ***c*** produces a *logical contradiction*.
Consider a spacecraft moving with the speed of light relative to the earth. So, one **observer O on the earth** would say; the spacecraft moves with speed ***c***. Let's suppose now that the spacecraft turns on a headlight. The observer O measures the speed of light beam and finds its value ***c***. So, *for observer O the spacecraft and the beam of light move togaether and will be all time at the same positions*. ***But, the second principle for spacecraft frame says that light travel with speed c relative to spacecraft; consequently they cannot be at the same position. Contradiction!***
This contradiction can be avoided only if one accept that it ***impossible*** for an *observer* O' in an *inertial* frame (***spacecraft, particle..***) to move with a speed equal to '**c**'.

Exercise: A high-speed space shift flaying with a speed c/2 past to a *land observer* O_1, sends out a pulse of light in all directions. Another *observer* O_2*, in the space shift*, measures a spherical wave front that spreads away with the same speed c in all directions. The observer O_1 makes his measurements. What is the propagation speed for him? ("***c"***) Does the wave front travel with the same speed in all directions?("***yes***") What is the shape of the wave front for him? ("**sphere"**) Is it centered on the space shift? ("***No***")

PRECISIONS ON THE MEASUREMENT PROCESS

- Einstein revised carefully all physics' basics and made the necessary corrections in relationship to his two postulates. At first, he ***considered carefully the concept of simultaneity***. This concept depends essentially from measurement procedure. That's why he decided to make some particular precisions of the procedure of measurement. He underlined that each *measurement action* is a particular **event** that occurs at a **single point** of space (x,y,z) at a **single instant** (t) in time as defined in a **given frame**. So, he characterized a physical measurement by an *event* with coordinates (***x,y,z,t***) in the inertial frame S. The ***same event*** has the coordinates (***x',y',z',t'***) in another inertial frame S'(moving with constant velocity versus S).

- He noted that a measure is made at a given point of space by an **observer** (person or device) that is **located** at this point. This observer can record only the events close to him and has to rely on his colleagues for events happening far from his location. So, to perform measurements that happen in *different locations* of **reference frame S a set of observers** is necessary. They are **distributed uniformly** in the frame S, they record events at their vicinity and transmit results to their colleagues. This way, all of them have the same information about what happens ***anywhere in frame S***. We name S, S',... a set of **inertial frames** defined as above.
We call **rest frame** the frame in which the ***object of the study is at rest***.

- Before starting the measurements, the observers in an inertial frame have to fix the procedures so that they can have the same conclusions. Essentially, they must select equal stick meters and equal watches. Then, they must *start 0 - moment of time in synchrony to each other*. How to realize this?

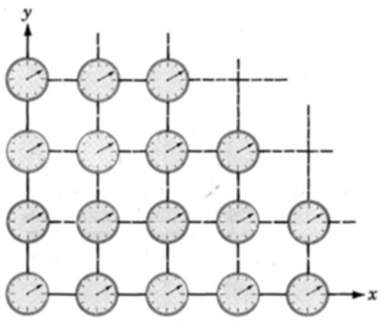


Fig 3

Einstein proposed the following scheme; Fix the distance between the adjacent observers equal to $\mathbf{3*10^8 m.}$ So, it will take **1s** for light to travel between them. By using this scheme, all observers can easily synchronize the moment t = 0 for their measurements. The first one (A) sends a light flash and the second (B) knows that its watch must be set at 1sec past, the third at 2sec past etc. By using this scheme of observers *in an inertial frame one is able to able to judge if two events happening in* ***different space locations*** *of an* ***inertial frame*** *are* ***simultaneous*** *or not.*

RELATIVITY OF SIMULTANEITY

- We are able to verify the simultaneity of two events that happen at the same instant close by an observer. Meanwhile, to be sure about the simultaneity of two events that happen at locations far from each other, we can use the upper *system of equidistant observers at rest in the same frame.* We will consider that there is one observer close to each event location.
Let's consider the case of observers A and B (at ***rest in frame S***) that explode ***simultaneously*** two firecrackers (fig. 4.a). The midway observer O in ***frame S*** receives two flashes ***simultaneously***.
To *declare a result we must clearly tell also the method used to get it*. So, we affirm that

Two events that happen at different locations of a given space are simultaneous if an observer in midway between them receives the flashes at the same instant.

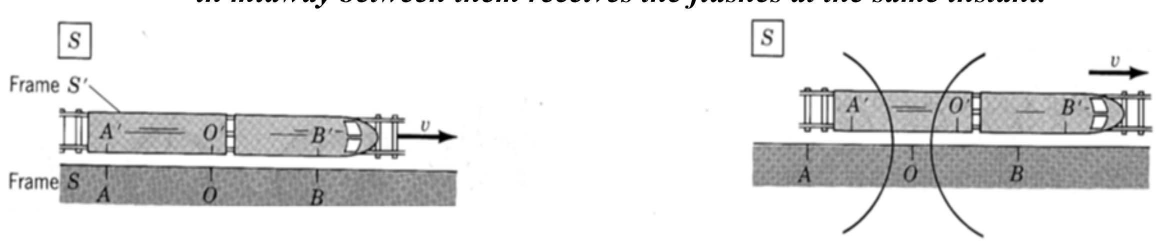


Fig 4 (a) (b)

- Let’s consider the same events in an ***inertial frame S’*** tied to a train moving with constant speed ***v*** with respect to frame S. Here we have to refer to front waves leaving A’, B’ observers and going versus O’ observer. As seen from the fig. 4.b, by the time the wave fronts have reached observer O, the observer O’ has moved on the right. Therefore the wave front from point B’ has already passed by O’ while the wave front from A’ has not yet reached O’. So, for the observer ***O’ in frame S’*** the explosions are ***not simultaneous***. This way, we get to the **relativity of simultaneity**:

The spatially separated events that are simultaneous in one frame are not simultaneous in any other inertial frame moving relative to the first one.

Note that this effect is entirely reciprocal; the events simultaneous in frame S’ will not be simultaneous in frame S.

Consequences

1) The time interval between two events is different in different frames (direct).
2) Even the lengths of an object measured in different frames are different (indirect).

TIME DILATION

-Let’s consider a ***thought experiment*** (fig. 5). A frame of reference S’ moves along a common direction to a frame S (axis Ox-O’x’) with relative constant speed ***v***. An observer O’ (in frame S’)

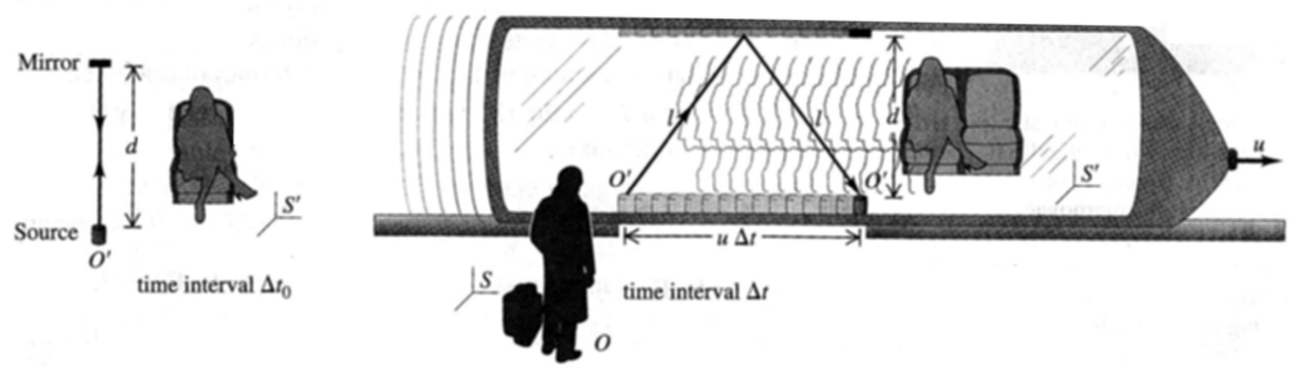


Fig5. (a) (b)

measures the time interval between two events that occur at the ***same point*** in space ***for*** its frame **S’**; A flash of light emitted by the source ER(emitting-recording) is reflected at the mirror M and captured by a receiver incorporated with the source at the same point ER. The time interval between these two events (emission-reception) in the ***rest frame*** S’ (fig 5.a) is:

$$\Delta t_0 = \frac{2 * d}{c} \qquad (21)$$

-Another observer O in the frame S finds a different interval of time between the same two events (emission-reception). As he ‘sees’ (fig. 5.b) that the two events happen at different locations in space (frame S), he observes that the length of round – trip path of light is:

$$2 * l = 2 * \sqrt{d^2 + (\frac{v\Delta t}{2})^2} \qquad (22)$$

Note that we have:

a) to use the same **speed of light** in he two frames – in accord to ***second postulate***.

b) to assume the same vertical distance '**d**' for the two observes. We will clarify this in the next section. Then, the time interval found by observer O is:

$$\Delta t = \frac{2*l}{c} = \frac{2}{c}\sqrt{d^2 + (\frac{v\Delta t}{2})^2} \qquad (23)$$

To find the relation between Δt and Δt_0, we transform this expression as follows

$$\Delta t = \frac{2}{c}\sqrt{d^2 + (\frac{v\Delta t}{2})^2} = \frac{2}{c}\sqrt{\left(\frac{c\Delta t_0}{2}\right)^2 + (\frac{v\Delta t}{2})^2} \rightarrow\rightarrow \left(\frac{c\Delta t}{2}\right)^2 = \left(\frac{c\Delta t_0}{2}\right)^2 + (\frac{v\Delta t}{2})^2 \rightarrow\rightarrow$$

$$\left(\frac{c\Delta t}{2}\right)^2 \left(1 - \frac{v^2}{c^2}\right) = \left(\frac{c\Delta t_0}{2}\right)^2 \rightarrow\rightarrow \Delta t\left(1 - \frac{v^2}{c^2}\right)^{1/2} = \Delta t_0 \_So,\_the\_final\_result\_is$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \qquad (24)$$

- Definition: If *two* ***<u>events</u>*** occur at the ***<u>same point</u>*** of a given ***<u>frame</u>*** we say that the measure is done in the ***<u>rest frame</u>***. The expression (24) tells that the **time interval** between the two **events in the rest frame** ***<u>Δt_0</u>*** is the **smallest possible**. For any other frame moving with constant speed ***v*** relative to the rest frame, the time interval the time interval between the two events ***Δt*** is given by (24) and it is always bigger than Δt_0. *<u>This effect is known as time dilation</u>*.

-The two following nominations are very familiar in theory of relativity

$$\beta = \frac{v}{c} \quad (25) \qquad \gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = \frac{1}{\sqrt{(1-\beta^2)}} \quad (26) \qquad \text{and} \qquad \Delta t = \gamma * \Delta t_0 \qquad (27)$$

Note that for 'normal speed values' ***v << c*** and ***γ = 1***. So, we find the same interval $\Delta t = \Delta t_0$ for all inertial reference frames (Newtonian approach). For high speed values, ***γ > 1. Note that β ≤ 1.***

-There is ***<u>only one frame</u>*** of reference in which ***<u>a clock</u>*** is ***at rest toward the object of study*** and there are infinitely in which it is moving. Therefore, the time interval measured between two events (such as two ticks of the clock) that occur at **the same point** in a particular frame is more fundamental quantity than the interval between the events in different points. *We use the term* ***proper time*** *to describe the time interval* Δt_0 *between* ***two events*** *that occur* ***<u>at the same point</u>*** *of a frame*.

LENGTH CONTRACTION

-Consider a rod AB *at rest* in frame S (fig.6). This is the ***rest frame*** for the rod, and its length in this frame is called ***proper length*** L_0. *The* ***proper length*** *of an object is the* ***space between its ends measured in the rest frame of the object***. Note that, to avoid discussion, one must define precisely the length *measurement procedure* and use it the same way for the two considered frames.

- Consider an observer **O'** moving with velocity "***v***" versus the laboratory frame. He does the measurements and calculations in frame **S'**. For his frame (**S'**), the rod moves with velocity "**- *v*** " while he, ***being at rest***, watches his watch. For this observer the two events ('1'- end A in front of him and '2'- end B in front of him) happen (fig.6b) at the *same space location* (he is **not moving versus S'**). So, he measures the **proper time** Δt_0. Then, he calculates the rods' length L'(**measured in S'**) as

$$L' = v^* \Delta t_0 \qquad (28)$$

This is **not** the rod's **proper length** because it is **not at rest** with respect **to this frame**.

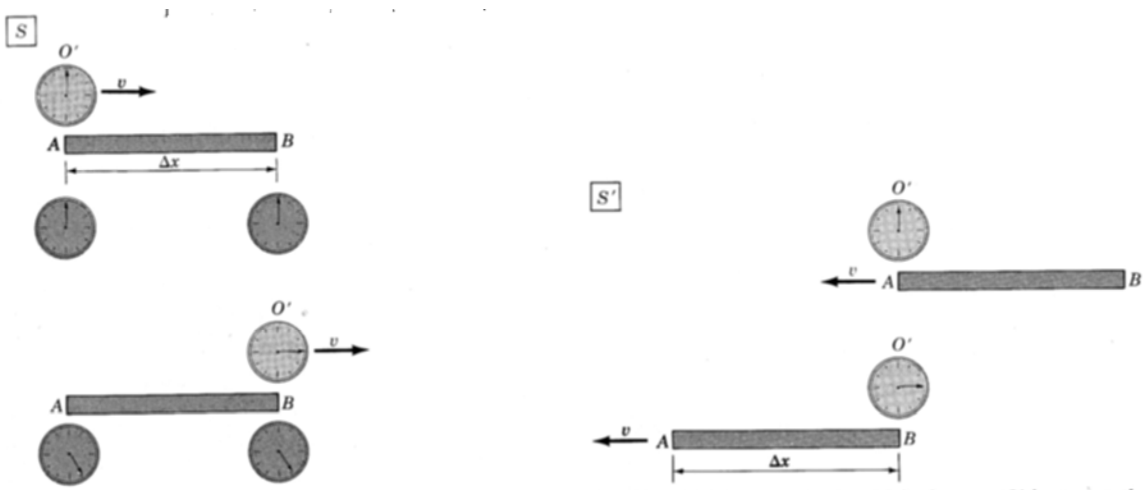


Fig 6 (a) (b)

-Two observers A and B in the ***frame S*** use the same method; They measure the interval of time Δt (**measured in S**) between the same two events: ('1'- end A in front of observer **O'**; '2'- end B in front of observer **O'**) and calculate the rod's proper length by multiplying the velocity ***v*** (of observer O') to the time interval between two events Δt (fig 7.a).

$$L_0 = v^* \Delta t \qquad (29)$$

-We know that the time interval Δt in an inertial frame is expressed through the proper time interval Δt_0 as $\Delta t = \gamma * \Delta t_0$. By this relationship, and (28-29) we relate L_0 and L'

$$\frac{L_0}{v} = \gamma \frac{L'}{v} \rightarrow\rightarrow\rightarrow\rightarrow L' = \frac{1}{\gamma} L_0 \qquad (30)$$

So, the measured length L of a moving object in inertial frames is

$$L = \frac{L_0}{\gamma} = L_0\sqrt{\left(1-\beta^2\right)} = L_0\sqrt{\left(1-\frac{v^2}{c^2}\right)} \tag{31}$$

As $\gamma \geq 1$, it comes out that $L \leq L_0$. This means that the object length measured in a moving frame is **smaller** than then its **proper length**. This effect is called ***length contraction***.

Notes:

a) The contraction effect is reciprocal. A similar object at rest in the frame S' will have a contracted length in the frame S.
b) *Only lengths parallel to the direction of motion are affected; those* ***perpendiculars*** *to the direction of motion are* ***not affected*** *by the contraction effect.*
c) This effect is ***not an optical illusion***. A 1m ruler is really shorter when measured in frames S' different from its rest frame S.