SOUND

GENERAL

- Sound: is a ***mechanical longitudinal*** wave;
  - that produces oscillations of density and pressure in the matter that propagates it;
  - that propagates in fluids (*liquids &gases*) and solids.

- The *main characteristic* of a *sound wave* is the ***frequency***. A mechanical longitudinal wave is classified as:
  A) *Sound wave* when its frequency is between 20 Hz and 20000Hz.
  B) *Infrasonic wave* when its frequency is smaller than 20 Hz.
  C) *Ultrasonic wave* when its frequency is bigger than 20000Hz.

- In general, the *amplitude* of oscillations is *very small compared* to equilibrium values for density or pressure.(Ex. Air pressure 1Atm = 10^5Pa while the amplitude of pressure oscillations is ~ 1Pa(= 1N/m^2).

-The *sound speed* is the *speed of longitudinal* mechanical *waves* in a medium which in :

a) a fluid is $v_L = \sqrt{\frac{B}{\rho}}$ B[N/m^2] - the "bulk modulus" is in the role of *restoring force*

b) a solid is $v_L = \sqrt{\frac{Y}{\rho}}$ where the Young [N/m^2] modulus is in the role of *restoring force*.

ρ[kg/m^3] - the density(fluid or solid) is in the role of *inertial factor* .

In the following, we consider the sound propagation in a fluid matter because it is easier to visualize the physical meaning of phenomena, but the results are equally valid for a solid.

17.1 NATURE OF SOUND

- Fluid in *equilibrium* means uniform density and pressure in the whole volume. There is only a *random* movement of molecules but the average of their displacement in time is zero. So, we model a fluid in equilibrium as constituted by molecules at rest at their equilibrium location. When a sound wave propagates in the medium, each molecule moves around its equilibrium position and along the direction of wave propagation.

- Suppose that the displacement of fluid molecules in a pipe at given moment ***t*** is that of fig 1-a. Then, the fig 1-b presents the pressure variations at different locations. The ***maxima of pressure*** variations are located ***between two consecutive maximum displacements with inverse sign***. Note that there is no pressure change at the locations with maximum displacement (+ or -). **General rule: In a sound wave, the pressure (and density) fluctuations are π/2 out of phase with displacements. Zeros in pressure *fluctuation* correspond to maxima or minima of particle displacements**.

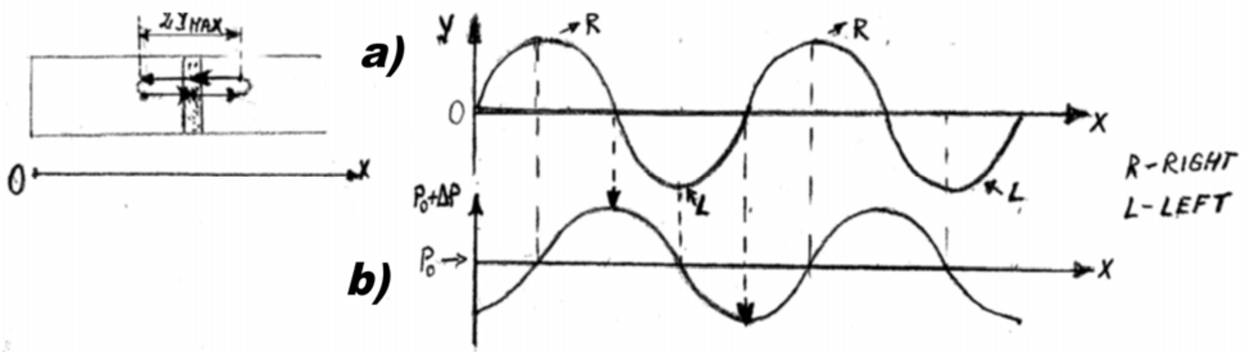


Fig 1 * * * *** * * * * * * **** * * * * * (*Molecules)*

-When dealing with 2D and 3D waves, the wave front t is a very important concept. The **WAVE FRONT** is the **locus of space points** at which the wave function has exactly **the same phase**. (Ex. The same crest of water surface). Spherical waves in uniform medium; it becomes practically a plane wave at big distance from the source.

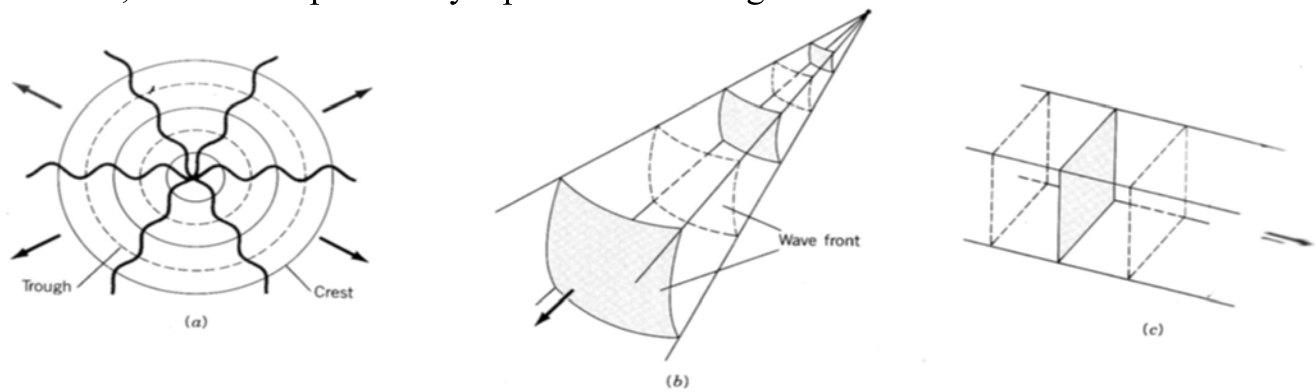


Fig 2

17.2 SOUND RESONANCE IN PIPES

- Remember what happens with *displacement* at the fixed and non fixed ends when a pulse propagates on a string. In the two cases there is a reflected wave at the end. This wave has the same frequency as the original one and travels along the opposite direction. These two waves interfere and build up a *standing wave on the string*. The resonance appears when ***L/λ*** fulfills a set of conditions. When dealing with resonance in pipes the same model applies.

-There are two types of pipes; **open** pipes (***both ends** are open*) and **closed** pipes (***one end** is closed)*. Taking into account the particular relation between the displacement and the pressure (see fig 1), one gets two criterions:

a) At a Closed end; there is a ***node** for the **displacement*** and an *antinode for the pressure;*

b) At an Open end; there is *atmospheric pressure which does not change in time*.

So, there is a *node for the pressure* and an ***antinode** for the **displacement***.

-Based on these criteria, and by referring to ***displacement***, the resonance appears;

a) for the **closed pipes** (see fig 3.a)

$$\lambda/4 = L \rightarrow v*T = 4*L \rightarrow 1/f = 4*L/v \Rightarrow\Rightarrow\Rightarrow f_1 = v/4*L$$

$$3\lambda/4 = L \rightarrow v*T = 4/3*L \rightarrow 1/f = 4/3*L/v \Rightarrow\Rightarrow\Rightarrow f_2 = 3v/4L$$

$$5\lambda/4 = L \rightarrow v*T = 4/5*L \rightarrow 1/f = 4/5*L/v \Rightarrow\Rightarrow\Rightarrow f_1 = 5v/4L$$

b) for the **open pipes** (see fig 3.b)

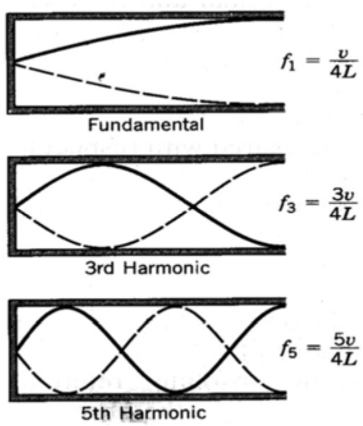
$$\lambda/2 = L \rightarrow v*T = 2*L \rightarrow 1/f = 2*L/v \Rightarrow\Rightarrow\Rightarrow f_1 = v/2L$$

$$\lambda = L \rightarrow v*T = L \rightarrow 1/f = L/v \Rightarrow\Rightarrow\Rightarrow f_2 = v/L$$

$$3\lambda/2 = L \rightarrow v*T = 2/3*L \rightarrow 1/f = 3/2*L/v \Rightarrow\Rightarrow\Rightarrow f_1 = 3v/2L$$

So; for <u>*closed pipes*</u> only ***odd*** harmonics are possible $f_n = \frac{nv}{4L};\_n = 1,3,5,7..$

for <u>*open pipes*</u> ***all*** harmonics are possible $f_n = \frac{nv}{2L};\_n = 1,2,3,4,5,..$



THREE FIRST NORMAL MODES OF A PIPE

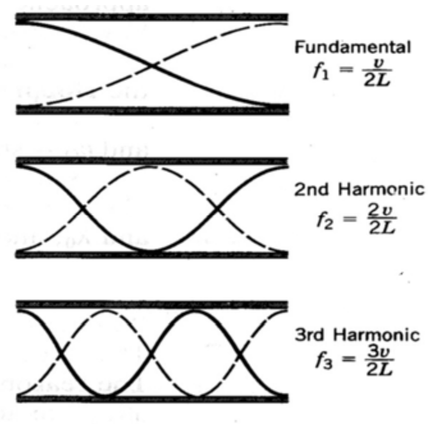


Fig 3.a fig 3.b

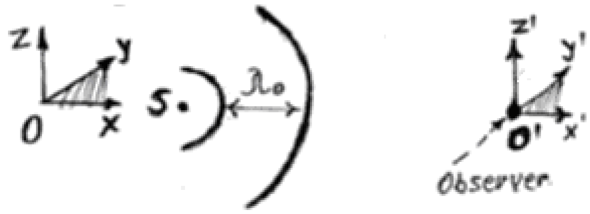
Consider that a sound wave comes into a pipe. If the sound ***frequency*** matches one of the normal mode ***frequencies***, there is a ***resonance*** produced into the pipe.

17.3 <u>DOPPLER EFFECT</u>

- The blowing horn of train at rest in station has its own frequency f_0. When the train approaches the station one hears higher frequency ($f_1 > f_0$). When the train leaves the station one hears lower frequency ($f_1 < f_0$).

- The frequency change due to a ***relative motion*** between the ***source*** of wave and ***observer*** *i*s called Doppler Effect. This phenomenon happens *for any kind of wave*.

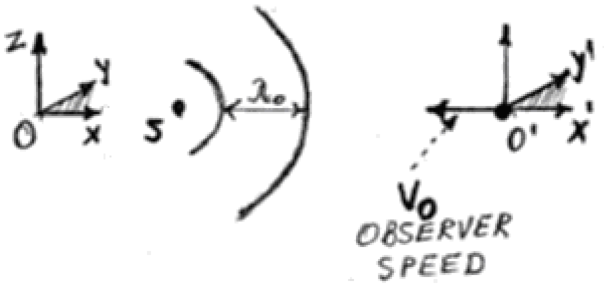
- Consider sound propagation in still (***at rest) air***. Consider two frames; ***Oxyz*** tied to ground "*at rest* to the air" and ***O'x'y'z'*** is tied to the observer. Let's call v the ***sound speed*** in still air, v_S the ***speed of source*** versus ground and v_O the ***speed of the observer*** versus the ground (***Oxyz*** frame). There are three different situations:



a) $V_S = 0$; $V_O = 0$ (train & observer at rest in the station). If the horn frequency is f_0 the wavelength of sound (in Oxyz) is $\lambda_0 = \frac{v}{f_0}$ (1)

b) $V_S = 0$; $V_O < 0$ (train at rest; observer approach the train). The system of wave fronts built in air (Oxyz) has wavelength λ_0 and when measured in O'x'y'z' the $\lambda' = \lambda_0$. The wave velocity in system O'x'y'z is $v + V_0$ and the sound frequency is

$$f_1 = \frac{v+V_0}{\lambda'} = \frac{v+V_0}{\lambda_0} = \frac{v+V_0}{v} f_0 \quad (2)$$



If $V_S = 0$; $V_O > 0$ (train at rest; observer

Fig 4

on the platform runs away from the train, right side). $f_1 = \frac{v - V_0}{v} f_0$ (3)

So, in general, $f_1 = \frac{v +/- V_O}{v} f_0$ (4) (+) Observer approach; (-) Observer moving away.

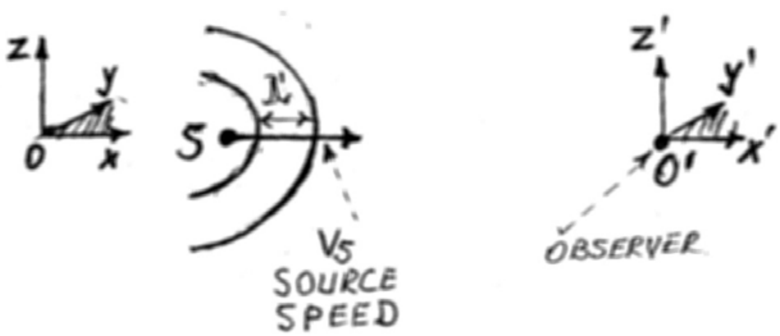


Fig 5

c) $V_S > 0$; $V_O = 0$ (train coming in the station; observer at rest). The frame O' is at rest versus frame O and the calculations for frame O are valid for O', too. In this case, during a period the wave front makes the path $s_{front} = v * T$ but in the same time the source makes the path $s_{source} = V_s * T$. So, at the end of ***one period***, the distance between source and wave front (i.e λ-wavelength) in the frame O(and O').

$$\lambda = \lambda' = vT - V_S T = (v - V_S)T = \frac{v - V_S}{f_0} \quad (5)$$

In general, for the frame O'x'y'z' we have $\lambda' = \frac{v}{f'}$

So, $\lambda' = \frac{v}{f'} = \frac{v - V_S}{f_0}$ and $f' = \frac{v}{v - V_S} f_0$ (6)

If $V_S < 0; V_O = 0$ (train moving away from the station, observer at rest).

We would have $$f' = \frac{v}{v + V_S} f_0 \qquad (7)$$

So, in general $$f_1 = \frac{v}{v +/- V_S} f_0 \qquad (8)$$ (-) Train approach; (+) Train moving away.

The general formulae for Doppler effect has the form:

$$f_1 = \frac{(v +/- V_0)}{(v +/- V_S)} f_0 \qquad (9)$$

a) Frequency increase
   1. Observer approach ($+V_0$, $V_S = 0$)
   2. Source approach ($V_0 = 0$, $-V_S$)
   3. Source and observer approach ($+V_0$ and $-V_S$)

b) Frequency decrease
   1. Observer move away ($-V_0$, $V_S = 0$)
   2. Source move away ($V_0 = 0$, $+V_S$)
   3. Source and observer move away ($-V_0$ and $+V_S$)

- In general, $V_0, V_S <$ ***v*** and frequency modifications are small.
Ex: If train approaches with $V_S = 50\text{Km/hrs} = 50*10^3\ \text{m}/3.6*10^3\text{s} = 13.9$ m/s, as $v = 344$ m/s, an observer at rest, will hear the frequency is $f_1 = \frac{344}{344 - 13.9} f_0 \approx 1.06 f_0$. So, there is only 6% increase in frequency.

17.4 <u>BEATS</u>

The principle of superposition is valid for all kind of waves (mech. & E.M.)

$$y_{Tot} = \sum_{i=1}^{n} yi\_\ for\_\ i = 1,2,\ldots.n \qquad (10)$$

This principle explains how two or more waves may produce interference in a given "region of space". It explains "standing waves", resonance, nodes and anti-nodes. These phenomena depend on the wave features in "space domain" (for different x,y,z-values).

- Let's see how the same principle can explain a very important physical effect "beats" produced by waves in "time domain"(same location but different t-values). Consider:
  - a given space location x = 0, to exclude the space effects;
  - two waves passing by x = 0 and producing "displacement" along the same direction (traverse or longitudinal);
  - having the same amplitude A;
  - having slightly different frequencies ($f_2 \cong f_1$);

The wave functions are $y_1 = A\sin(\omega_1 t)\_\omega_1 = 2\pi f_1$ $y_2 = A\sin(\omega_2 t)\_\omega_2 = 2\pi f_2$

-By use of superposition principle at point x = 0 we get its displacement as

$$y_{Tot}(x=0) = y_1 + y_2 = A\sin(\omega_1 t) + A\sin(\omega_2 t) = 2A\sin\left(\frac{\omega_1 + \omega_2}{2}\right)t * \cos\left(\frac{\omega_1 - \omega_2}{2}\right)t \text{ and}$$

$$y_{Tot}(x=0) = 2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t * \sin 2\pi\left(\frac{f_1 + f_2}{2}\right)t \qquad (11)$$

Note that the expression (11) describes the ***time behavior*** at a ***given point*** in space.

- Suppose that $f_1 > f_2$. In this case $\frac{f_1 - f_2}{2}$ is a very small frequency and the factor $2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t$ is ***in the role of amplitude*** that *varies slowly* in time while the factor $\sin 2\pi\left(\frac{f_1 + f_2}{2}\right)t$ presents a wave oscillation with frequency very close to $f_1\_and\_f_2$.

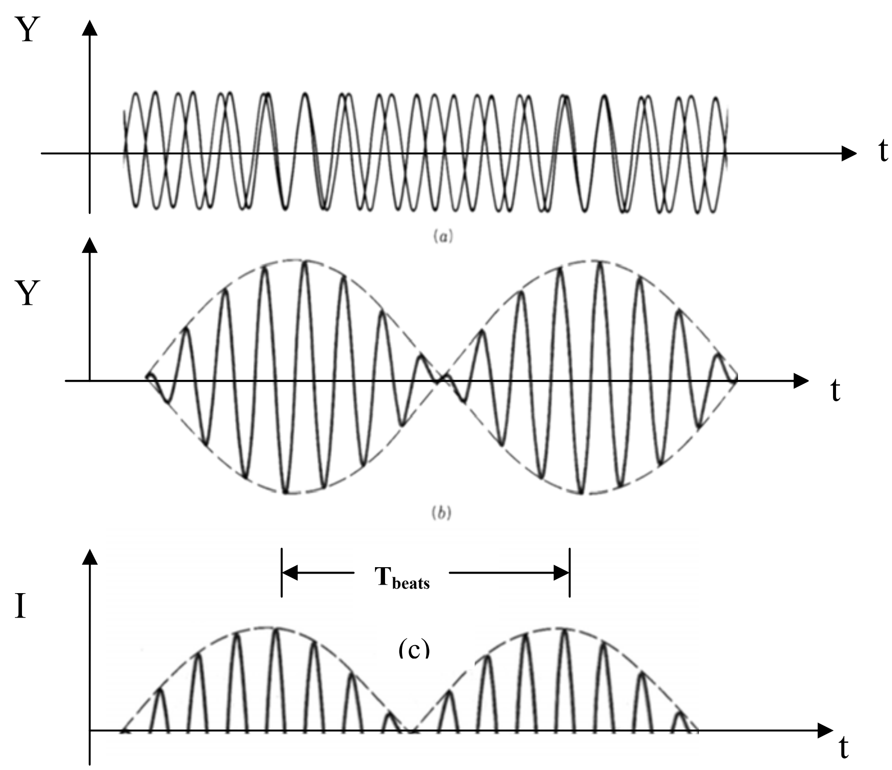


Fig. 6

-This wave is shown in figure 6.b. It represents "***wave interference in time domain***". This kind of amplitude variation is characteristic for a phenomenon called ***wave "BEATS***". Note that we hear the intensity (loudness) of sound waves that is proportional to the square of graph b in figure 6b. As the intensity is only the positive (fig.6c), we perceive regular sound maxima " beats" with twice frequency $f_{BEATS} = 2 * \frac{f_1 - f_2}{2} = f_1 - f_2$. (12)

17.6 SOUND INTESITY

To get a better meaning of the sound ***intensity*** we will start with the calculation of *power transported* by a TW mechanical *wave* on a string and a sound wave in a pipe.

A] THE POWER TRANSPORTED BY A TW TRAVELLING ON A STRING

Consider that a point of string (***wave source***) is moved *up* and *down* with amplitude Y_{max} and a *frequency* ***f***. We chose a reference *frame* with *origin O at source* and directed along the direction of wave propagation (fig.7). Any string point (***small element with length dx***) will

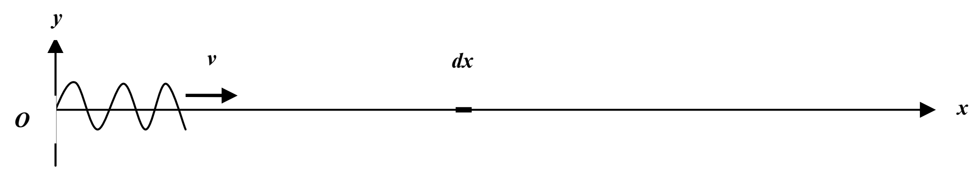


Figure 7

oscillate with amplitude Y_{max} (***assuming no energy loss***) and the same frequency ***f***. If the *linear density* of the string is μ, the mass of a "*string point*" with *length dx* is $dm = \mu dx$ (13)
The oscillations' energy of this "point" is given by (14) and the power passing by this point

$$dE = \frac{1}{2}kY_{\max}^2 = \frac{1}{2}(\omega^2 dm)Y_{\max}^2 = \frac{1}{2}\omega^2 Y_{\max}^2 dm = \frac{1}{2}\omega^2 Y_{\max}^2 \mu dx \quad (14)$$

$$P = \frac{dE}{dt} = (\frac{1}{2}\omega^2 Y_{\max}^2 \mu)\frac{dx}{dt} = \frac{1}{2}\omega^2 Y_{\max}^2 \mu v = \frac{1}{2}(2\pi f)^2 Y_{\max}^2 \mu v \quad (15)$$

i.e. the power transported by wave is given by expression (15) where **v** is the *wave speed*.

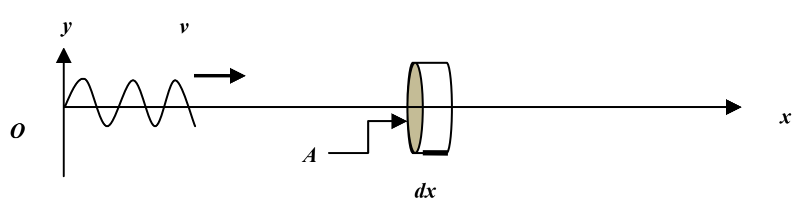
Example: The ***power*** transported by a TW with amplitude ***2mm*** and frequency ***50Hz*** that propagates at ***10m/s*** on a string with linear density ***10g/m*** is

$$P = 0.5*(2*3.14*50)^2*(0.002)^2*(10*10^{-3})*10 = 1.97*10^{-2} W.$$

Note: This model and its result (expr.15) are valid for the power transported by ***LW***, too.

B] THE POWER TRANSPORTED BY A SOUND WAVE (LW) IN A PIPE

Let's apply now this model for calculation of power transported by a sound wave (***LW***) when propagating inside a ***pipe***. Assume that a sound wave with constant ***displacement amplitude*** Y_{max} and **frequency *f*** propagates inside a ***pipe*** with transverse ***area A.*** We need only to correct the expression for the "*oscillating mass*". Now, it is the mass inside the cylinder with small length ***dx***. If the *volume density of mass* is $\rho[kg/m^3]$, then $dm = \rho dV = \rho A dx$ *(16)*



Then, the two expressions (14-15) transform to

$$dE = \frac{1}{2}\omega^2 Y_{\max}^2 dm = \frac{1}{2}\omega^2 Y_{\max}^2 \rho A dx \quad (17)$$

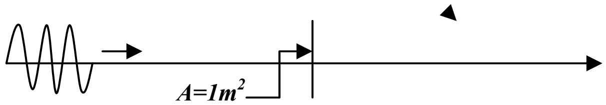
$$P = \frac{1}{2}(2\pi f)^2 Y_{\max}^2 \rho A v \quad (18)$$

Example: The ***power*** transported by a sound wave with frequency ***50Hz*** and ***displacement amplitude 1mm*** inside ***a pipe*** *with* ***radius 2cm*** *filled with* ***air (v =340m/s, ρ =1.29kg/m3) is***

$$P = 0.5*(2*3.14*50)^2*(0.001)^2*(1.29)*[3.14*(0.02)^2]*340 = \mathbf{0.027W}$$

C] DEFINITION OF WAVE INTENSITY. INTENSITY OF A SOUND WAVE

A sound wave transports energy along its direction of propagation. The *intensity* ***I*** of this wave is "the ***amount of power (w =J/s) that falls on a unit area (w/ m²) perpendicular to its direction of propagation "***



$$I\ (\mathrm{W/m^2}) = P/A \qquad (19)$$

In case of example given upside $I = 0.027W/ 3.14*(0.02)^2 = 21.49 W/m^2$

Note : This definition for intensity is valid for any kind of wave no matter its type or physical nature.

- The human ear is able to hear sound waves in the range [10^{-12} **to 1 W/m²**] but it does not have a linear response to the intensity (example *it does not perceive 2 times louder a sound of 1 W/m² when compared to a sound of 0.5 W/m²*). The experiments show that the human ear reacts in a logarithmic way versus the ratio of two sound intensities and this response is described pretty well by the relation:

$$\beta = 10\log_{10}\frac{I}{I_o} \qquad (20)$$

-As a matter of fact, the coefficient ***β has no units*** but for practical purposes it is called the *level of intensity* in ***decibel*** (dB) units. To be precise, this requires fixing a reference value for I_0. For the sound, by convention $I_0=10^{-12}\ W/m^2$. This way, the

hearing threshold is $$\beta = 10\log_{10}\frac{I_0}{I_o} = 10\log_{10}1 = 0dB \qquad (21)$$

pain threshold is $$\beta = 10\log_{10}\left(\frac{1W/m^2}{10^{-12}W/m^2}\right) = 10\log_{10}10^{12} = 12*10\log_{10}10 = 120dB \qquad (22)$$

Common Noise Sound Levels

- 30 db - Whisper ; 60 db - Normal conversation ; 80 db - Ringing telephone
- 90 db - Hair dryer, power lawn mower ;98 db - Hand drill ; 105 db - Bulldozer
- 110 db - Chain saw ; 120 db - Ambulance siren ; 140 db - Jet engine take-off

For example given upside $\beta = 10log(21.49/10^{-12}) = 10log(21.49*10^{12}) =$
$= 10log(21.49) + 10log10^{12} = 13.3+120 = 133.3dB$

- The emitted power may be ***concentrated*** around ***one direction*** or ***distributed uniformly*** in space. The first case concerns the majority of technical applications of sound waves but it is more complicated to study. In this course, we will consider only the second case where the energy is propagated by ***uniform plane waves*** or spreads *uniformly over spherical wave fronts*. In this ***last case***, if the sound wave is produced with ***power P at its source*** and does not get absorbed, at a ***distance R*** *from the source*, the sound wave will have the ***intensity***

$$I = \frac{P}{4\pi R^2} \qquad (23)$$