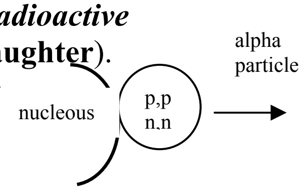
RADIOACTIVITY

- From about **1500** known **nuclides**, there are only **250**, all with **Z ≤ 83 that are stable**, but not any nuclide with Z < 83 is stable.. What happens with the 1250 others nuclides?
After existing for a certain time, the primary nuclide (***parent***) emits spontaneously a "***radioactive emission***" i.e. ***α - particles or β - particles*** and transform to another type of nuclide (**daughter**).
The **α -particle** is a **doubly ionized He atom**; i.e. $({}_{2}^{4}He)^{++}$ with electric charge q = +2
The ***β - particles*** are ***electrons*** ($m = m_e$, q = -e) or ***positrons*** ($m = m_e$, q = +e).
Most of the ***radioactive elements*** emit ***either α or β*** particles but ***a few emit both***.
In general, the ***daughter nuclei*** created after the emissions of α or β particles are ***in excited energy*** levels but they drop very ***quickly*** to the ***ground level***. This "secondary process" is accompanied by the **emission** of excitement energy in the form of a ***γ-radiations***. The ***γ-radiations*** are considered as a ***radioactive emission***, too. These ***quantum particles*** are **photons** (m = 0, q = 0) with ***very high energy***. The corresponding waves in electromagnetic spectrum have shorter wavelength than X rays.



- Like any other physical process, a radioactive transmission must obey to the "**principle of energy conservation** " the sum energy of the products must be equal to the energy of the parent nuclide".
We tie the calculation frame to the parent nuclide; so its energy is just the rest energy

$$E_{initial} = m_{0(parent)} * c^2 \qquad (1)$$

Meanwhile the ***products*** of radioactive emission ***move*** with respect to parent nuclide; they have ***rest and kinetic energy*** in selected frame. So, their energy is $E_{final} = \sum_i m_0^i * c^2 + K$ (2)

where the second term contains the ***sum of kinetic energy of all the products***. From the principle of energy conservation we have $m_0(parent) * c^2 = \sum_i m_0^i c^2 + K \rightarrow m_0(parent) * c^2 > \sum_i m_0^i c^2$ and

$$m_0(parent) > \sum_i m_0^i \qquad (3)$$

The relation (3) is ***basic requirement*** for any ***radioactive process***. It means that the total mass of the products must be smaller than the mass of parent nuclide. This requirement explains "*Why the natural radioactivity cannot produce other types of free particles"*; say.. a single proton (or neutron) or five protons (or neutrons),…? The requirement is not fulfilled in those other cases…..

The ***energy released*** in ***each radioactive event*** is called ***disintegration energy*** Q. So

$$Q = [m_0(parent) - \sum_i m_0^i] * c^2 \qquad (4)$$

This energy is the sum of the kinetic energy of particles (daughter nuclide, α , β) plus the energy of released gamma radiation that appears after one radioactive event.

-A value ***Q < 0*** tells only that the process is ***impossible by itself***. But, if one ***provides*** this ***energy*** to the ***parent nucleus, the energy restriction is overcome and*** it can be ***disintegrated*** the same. If the nuclide disintegration happens this way we call it a ***nuclear fission reaction. The provided energy** is called* the ***separation energy*** (*noted **S***) of ***fission reaction*** and it is calculated as ***S = - Q*** .

Remember : "*radioactivity is an natural spontaneous phenomenon" which means that a decay event* **happens with a certain probability within a time interval** *and not a given moment of time.*

ALPHA DECAY

- Almost all **unstable** nuclides with **A > 150** undergo **α -decay**. With a few exceptions, the nuclides with **Z < 83** do **not** undergo **α-decay**.

During an **α-decay** event, the **parent** nucleus ${}_{Z}^{A}X$ emits **one** α - particle (He^{++} i.e. the nucleus of the helium atom) . This process decreases its charge form **+Ze** to **+(Z - 2)e** and the mass number from **A** to **A-4**. So the relation that describes an α-decay event is:

$$ {}_{Z}^{A}X \Rightarrow \alpha - decay \Rightarrow {}_{Z-2}^{A-4}Y + ({}_{2}^{4}He)^{++} \quad (5)$$

Ex. (see Fig.1): ${}_{88}^{226}Ra \Rightarrow \alpha - decay \Rightarrow {}_{86}^{222}Rn + ({}_{2}^{4}He)^{++}$

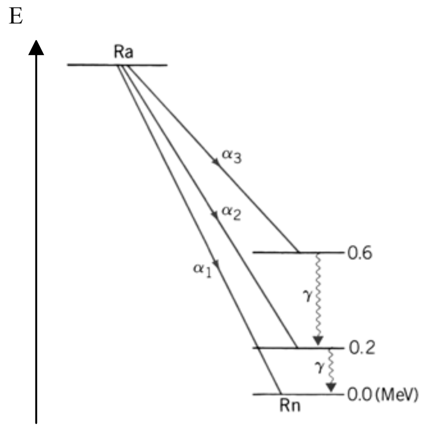


Figure 1

The **radium** nuclide in ground state emits one **α-particle** and it is transformed into a **radon** nuclide. Due to the three energy levels of radon nuclide (ground & two excited levels), there exist three possibilities for the kinetic energy of emitted α- particle. The ***maximum energy*** corresponds to the shift between the two respective energy ground levels. It is ~***4.8MeV***. Each of two transitions to excited levels of radon nuclide are followed by additional de-excitement shifts to lower energy levels through emission of a **γ- particle**.

So, during an α-decay process produced in a sample containing Ra nuclides, one would expect to record:

- The daughter nuclides Rn at different energy[1] levels;
- α particles with energy ~4.8 MeV, ~ 4.6MeV, ~ 4.2MeV
- γ particles with energy ~0.2MeV, ~0.4 MeV, ~0.6 MeV

Note: ***The γ "particles" that follow the α-decay have a discrete spectrum of energies.***

What is the ***disintegration energy of*** *this* ***reaction***? At first, we note that no matter what kinetic energy has the α-particle, the disintegration energy of radioactive event **is the same**. So, we refer to the simplest scheme, α_1-particle and ${}_{86}^{222}Rn$ in ground level of energy. By applying the relation (4), we get

$$Q = [m_0({}_{88}^{226}Ra) - m_0({}_{86}^{222}Rn) - m_0({}_{2}^{4}He^{++})] * c^2 = [m_0({}_{88}^{226}Ra) - m_0({}_{86}^{222}Rn) - m_0({}_{2}^{4}He^{++}) + 226m_e - 226m_e] * c^2$$

$$Q = [m_{atom}({}^{226}Ra) - m_{atom}({}^{222}Rn) - m_{atom}({}^{4}He)] * c^2 = (226.025408 - 222.01757 - 4.002603) * 931.5 \approx 4.9MeV$$

BETA DECAY

-During a β-decay the **parent** nucleus ${}_{Z}^{A}X$ emits a β- particle. There are **two kinds of β particles**: β^-, which are simply **electrons** ($m = m_{el}$, $q = -e$) and β^+, which are **positrons** ($m = m_{el}$, $q = +e$). The **positron** is an example of ***antimatter particles***. The antimatter particles appeared at first as theoretical models during the development of quantum mechanics. Then, several were observed experimentally.

- So, **two** possible **schemes** may be happen during **a β- decay** event:

[1] Here we refer to the energy levels of an atom "at rest". But, during an α-event the daughter nucleus gets some kinetic energy, too. In this case the Rn nucleus gets a recoiling kinetic energy ~ 0.1MeV which shift its ground level to 4.81MeV.

a) **β⁻ decay**. $Z \Rightarrow \beta^- \Rightarrow Z+1$ The ***daughter*** nuclide has ***one more proton***.
b) **β⁺ decay**. $Z \Rightarrow \beta + \Rightarrow Z-1$ The **daughter** nuclide has ***one proton less***.
Note that ***neither electrons nor positrons are particles that exist inside the nucleus***. So, it comes out that these emitted particles are ***created in nucleus during the process of β-decay***.

- By experimental measurements, it is proved that, the ***kinetic energy*** *of the emitted* **β- particles** ***is not discrete*** while ***the parent*** *and the* ***daughter*** *nuclides have* ***only discrete*** *energy levels*. This was an contradiction to the basic principle of energy conservation.
Fermi resolved this situation by ***predicting*** that together with the β ***-particle*** is emitted ***another particle that he called neutrino*** (***$m_{rest} \cong 0$, $q = 0$***). This particle (detected experimentally in 1956) transports one part of released energy and *interacts very weakly with matter* (weak interaction).
Remember that:
a) β-decay brought to the discovery of two "new" particles (***positron*** and ***neutrino***).
b) the ***energy of β-particles*** is ***continuous***.

-The actual scheme of a **β⁻** decay event is:

$${}^{A}_{Z}X \Rightarrow \beta^- decay \Rightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu} \qquad \bar{\nu} \text{ is an } \textbf{\textit{antineutrino}} \qquad (6)$$

or

$${}^{A}_{Z}X \Rightarrow \beta^+ decay \Rightarrow {}^{A}_{Z-1}Y + e^+ + \nu \qquad \nu \text{ is a } \textbf{\textit{neutrino}} \qquad (7)$$

Exemples: ${}^{14}_{6}C \Rightarrow \beta^- decay \Rightarrow {}^{14}_{7}N + e^- + \bar{\nu}$ ${}^{13}_{7}N \Rightarrow \beta^+ decay \Rightarrow {}^{13}_{6}C + e^+ + \nu$

- Note that *during β- decay event, the* ***mass number A*** *of the* ***daughter*** *nucleus (Y) is the same as* ***mass number A*** *the parent nucleus(X)*. The scientists explained this fact by considering that **β⁻ decay** is due to an ***initial transformation*** of one ***neutron to proton*** inside the parent nuclei following the scheme

$$n \Rightarrow p^+ + e^- + \bar{\nu} \qquad (8)$$

Then, the electron and antineutrino leave the nucleus while the proton remains inside.
The **β⁺** decay is due to an ***initial transformation of one proton to neutron*** inside the parent nuclei following the scheme $p^+ \Rightarrow n + e^+ + \nu$ (9)

Notes:
- **β⁻** decay showed that *neutron* and *proton* are *not true fundamental particles;* they do *disintegrate.*
- Since the rest mass of neutron is ***bigger*** than rest mass of proton, the process (8) may happen even in free space. But, the process (9) is not allowed to happen in free space by condition (4). It can be realised only inside the nuclides ***if the Q-value*** of considered nuclear ***reaction*** is positive.

GAMMA RAYS

-These are high-energy ***photons*** (***1KeV*** to several ***MeV***) that generally ***accompany α and β decay***.
But a nucleus can emit γ rays even without being transformed to another type of nuclide. Actually, this type of radiation is emitted during the *energy de-excitement process of nuclei*. Since the nuclei energy levels are discrete, these ***photons have discrete spectrum of energy***.

THE DECAY LAW

- This is a ***statistical law*** of nature, i.e. knowing that a given nuclide is radioactive, it's ***impossible*** to predict *when precisely* it will decay but one may *predict the probability* that it decays during a *time interval.* When considering big sets of radioactive nuclides[2] the probabilistic behaviour appears in a simpler frame. If ***N*** is the total number of radioactive nuclides at the moment ***t*** and ***dN*** the number of those that suffer decay during the interval ***dt,*** the following relation holds on

$$\frac{dN}{dt} = -\lambda N \qquad (10)$$

The ***decay constant*** λ [1/sec] has a **characteristic** value for ***each specific*** radioactive *nuclide*.
The ***decay rate*** $R = -\frac{dN}{dt}$ informs about the ***number of decays*** happening in a ***unit of time*** (1sec).

-One integrates the equation (10) as follows

$$\frac{dN}{N} = -\lambda dt \rightarrow \int_{N_0}^{N} \frac{dN}{N} = \int_0^t -\lambda dt \Rightarrow\Rightarrow \ln N(t) - \ln N(0) = -\lambda t \rightarrow \ln \frac{N(t)}{N(0)} = -\lambda t \Rightarrow N(t) = N(0) * e^{-\lambda * t}$$

So, the number of ***remaining radioactive nuclides*** in sample, ***N(t),*** decreases with time as

$$N(t) = N(0) * e^{-\lambda * t} \qquad (11)$$

where ***N(0)*** is their number at the moment ***we start counting*** .

- Some important ***characteristic parameters*** of radioactive phenomena are:

a) ***Half time*** ($T_{½}$) = time required to have ***N(t) = N(0)/2***. One may calculate simply that

$$\frac{1}{2} N(0) = N(0) e^{-\lambda * T_{1/2}} \rightarrow -\ln 2 = -\lambda T_{1/2} \rightarrow \lambda T_{1/2} = 0.693 \quad \text{and}$$

$$T_{1/2} = \frac{0.693}{\lambda} \qquad (12)$$

b) The ***evolution*** of ***decay rate*** in time $R(t) = -\frac{dN(t)}{dt} = \lambda * N(0) e^{-\lambda t} = R(0) e^{-\lambda t}$ (13)

where the ***initial decay rate*** is $R(0) = \lambda * N(0)$ (14)

Notes:

- The ***decay rate units*** are Becquerel [***1Bq = 1decay/sec***] and Curie [***1Ci = 3.7*10^10 Bq***].
- If the *half – period is too big* (***$T_{½}$***) the *decay constant is very small* (λ ≈ 0), $e^{-\lambda t} \cong 1$ and the laboratory measurements give a **constant value** for the ***decay rate*** $R(t) = R(0) = \lambda * N(0)$.

[2] Which is the situation in a real experiment.

RADIOACTIVE DATING

-***Basic idea***; *the **relative abundance** of two isotopes in a sample, **one stable** and **one radioactive**, **changes with time** in a determined way due to radioactive decay **parameters** of the radioactive one*.

Critical requirement; One has to select a radioactive isotope with such $T_{½}$ that the ratio of isotopes' ***abundances changes*** in a ***measurable way during the time interval*** one wants to estimate[3].

-The ***natural processes*** are often conveniently ***selective*** as to what ***elements*** they do incorporate during the ***formation of materials***. ***In the simplest case, the material in its initial solidified form, i.e. at time t = 0 of its history as such, will incorporate a parent nuclide and reject the daughter nuclide***.
So, no daughter nuclides at all at t = 0. With this assumption, it comes out that *all the daughter nuclides present in a sample are "born" inside material by radioactive decay of parent nuclides starting from the moment the solidified sample was formed*.
If the material incorporates both the parent and daughter nuclides " at t = 0" , a correction must be made for the *initial proportion* of the *parent radioactive nuclide* and its *daughter*.

The two main applications of radioactive dating concern the *measurement of the* ***time elapsed since***:
a) the rock(***on Earth or Moon***) are formed b) a living organism has died

a] To measure the ***long*** *age of rocks* by radioactive dating one has to use ***long live*** *radioactive nuclides* like $^{40}_{19}K$ or $^{238}_{92}U$. The ***radioactive nuclide*** $^{40}_{19}K$ decays with $T_{1/2} = 1.25*10^9$ ***years*** to the ***stable nuclide*** $^{40}_{18}Ar$ ($^{40}_{19}K \rightarrow \beta^+ + ^{40}_{18}Ar$) .The ***radioactive nuclide*** $^{238}_{92}U$ decays (through a chain of radioactive events) to ***the stable nuclide*** $^{206}_{82}Pb$ with $T_{1/2} = 4.5*10^9$ ***years.***

Considering that all the $^{40}_{18}Ar$ inside the rocks[4] is due to radioactive ***β^+ decay*** of $^{40}_{19}K$, one estimates the rock age by use of the abundance ratio (N_{Ar} / N_K). Similarly, assuming that all of the lead in a rock is produced by the decay of original uranium one may estimate the rock age by use of ratio (N_{Pb} / N_U).

b] The *carbon atom in air* is met normally inside the molecule of CO_2. The measurements have shown that there is just one ***radioactive nuclide*** $^{14}_{6}C$ ($T_{1/2} = 5730$ ***yrs***) in $\sim 10^{13}$ **stable *nuclides*** $^{12}_{6}C$ *in air*. As the number of $^{14}_{6}C$ decreases with time (due to β^- decay to its ***daughter*** $^{14}_{7}N$; $^{14}_{6}C \rightarrow \beta^- + ^{14}_{7}N$), one would expect the ***ratio*** ($^{14}_{6}C/^{12}_{6}C$) to ***change in time***. But, the experimental measurements show that this ratio ***in air*** is practically ***the same*** *for a very long time. This* is explained by the *continuous production* of $^{14}_{6}C$ *isotopes* during the bombardment of ***N atoms*** by ***cosmic rays*** in the upper atmosphere.

-A ***living organism*** keeps a ***characteristic ratio*** ^{14}C / ^{12}C ***as long as it is living*** because of continuous exchange of CO_2 with air (***photosynthesis, breathing***). This ***ratio starts to decrease*** from the moment the organism dies (***^{14}C decreases due to its radioactivity while ^{12}C remains the same***). So, one can estimate the *time interval elapsed since* the ***organism*** died by comparison of its ratio ***$^{14}C / ^{12}C$*** to that of a similar living organism. This procedure is known as ^{14}C / ^{12}C ***radioactive dating***.

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[3] Most radioactive isotopes have short half-lives and lose their radioactivity within a few days or years.
[4] The daughter isotopes get trapped inside the rock material after solidification.