THE STRUCTURE OF NUCLEUS

- The atom consists of a nucleus and a number of electrons moving in atomic orbits (electronic clouds). The atom is electrically neutral. So, the nucleus contains an electric charge equal to the sum of electrons' charge but with positive sign. The forces between the nucleus and the electrons are electromagnetic forces while the rules that define the electron orbits are quantum rules. But, the orbital electrons" **don't know**" what happens inside the nucleus or the nuclear forces.

- The atom nucleus consists of ***nucleons*** (***protons and neutrons***). The proton has **e+** electric charge (electron charge **e-**). Consequence: There is the **same number of protons** (in nucleus) and electrons (in orbits) inside an atom (electrically neutral).

- Each ***element of periodic table*** shown by ***symbols*** (H, He,..) is identified by its ***atomic number***, **Z** = number of protons in nucleus. For **natural** elements Z = 1(H) to 92(U). The **artificial** elements have Z = 93-107. The ***atomic mass number*** A is calculated as

$$A(\text{nucleon nb.}) = N(\text{neutron nb.}) + Z(\text{proton nb.}) \qquad (1)$$

- The notation ${}^{A}_{Z}X$ indicates a ***nuclide*** (nucleus with ***precise number of protons and neutrons***). Examples ${}^{16}_{8}O, {}^{14}_{7}N$,..This notation is important for distinction of different ***isotopes*** (***A different***) of the same element (***same Z***). Since the atomic electrons determine the chemical proprieties, the ***isotopes are chemically identical***. Three isotopes of hydrogen are known (fig 1); hydrogen (H), deuterium (D), and tritium (T). The nuclei of all three isotopes contain *one proton*, which characterizes them as three forms of the same element; **hydrogen**; in addition, the deuterium nucleus has one neutron and the tritium nucleus has two neutrons. In the three forms, the neutral atom has one electron outside the nucleus that balance the positive charge of the single proton.

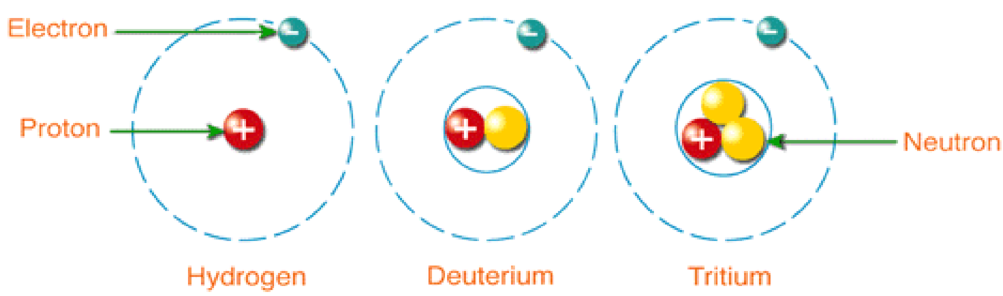


Fig 1

-The mass of a **nucleon** is ~$\mathbf{10^{-27}}$ **kg**, i.e. 10^4 times bigger than **electron mass (~$\mathbf{10^{-31}}$ kg)**. So, the ***mass of an atom*** is practically defined by the **number of nucleons** it contains "A". One uses "A" to express the atoms' mass by a proper mass unit at "atomic dimensions"; the ***unified mass unit-u***. This unit is ***defined*** as 1/12 part of the mass of the carbon isotope ${}^{12}_{6}C$; so $m_{{}^{12}_{6}C} = 12u$. One may find (Ex. 43.1) that ***$1u=1.66056*10^{-27}$ kg = 931.5 Mev/c^2***. The atomic masses given in **u-units** in the periodic table are the ***weighted averages over all isotopes*** of the element. It's useful to remember that

$$m_p = 1.67264*10^{-27}\text{kg} = 1.007276\ u = 938.28\ \text{MeV}/c^2$$
$$m_n = 1.6750*10^{-27}\text{kg} = 1.008665\ u = 939.57\ \text{MeV}/c^2$$
$$m_e = 9.109*10^{-31}\text{kg} = 0.000549\ u = 0.511\ \text{MeV}/c^2$$

-The radius of the smaller atom (first orbit in Bohr's model H) is ~0.0526nm = $5*10^{-11}$ m. The experiments show that nuclei have *almost spherical form* with radius

$$R \sim 1.2*A^{1/3}\ \text{fm} \qquad 1\text{fm(fermi)} = 10^{-15}\ \text{m} \qquad (2)$$

By using expression (2) for H nucleus we get $R_H \sim 1.2*1^{1/3}$ = 1.2fm which is 4 orders of magnitude smaller than the smallest H-atom orbit. Note that practically all atom mass is concentrated inside the nucleus; So, big values of mass density (Ex. 43.2 shows $\rho_M \sim 10^{17}$kg/m3 for ${}^{16}_{8}O$). From the relation between mass and energy one may infer that energy density has very big values inside the nuclei.

($E = m*c^2$ So, $\rho_E = E/V = (m/V)*c^2 = \rho_M*c^2$)

BINDING ENERGY AND NUCLEAR STABILITY

Introduction of binding energy concept

a) *Consider the* ***system earth plus two objects****; a magnet with masse m_1 fixed at a space location and iron with masse m_2 at the same height "h" in two different situations;*

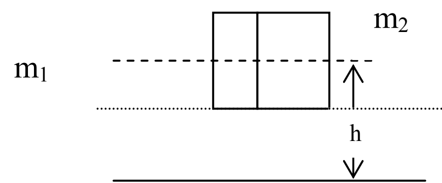
a-1) Iron object bound to the magnet.

The energy of the system is

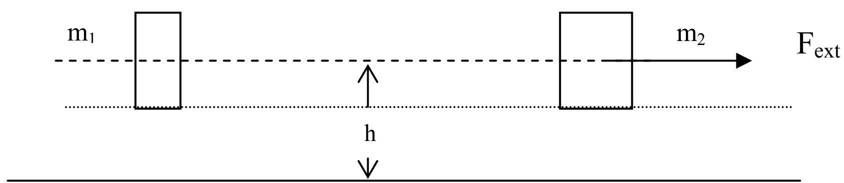
${}^1E_{sys}=U_1+U_2+E_{12}$

$U_1=m_1*g*h;\ U_2=m_2*g*h$

E_{12} is the magnetic binding energy



a-2) Iron object shifted far away from the magnet. Using an ***external force F_{ext}****, one detaches m1 from magnet and shifts it far away (no magnetic interaction m_1-m_2).*



The energy of the system is (no interaction m_1-m_2 means $E_{12}=0$)

${}^2E_{sys}=U_1+U_2$ *where* $U_1=m_1*g*h;\ U_2=m_2*g*h$

The positive work done by force F_{ext} to shifts the object m_2 far away from m_1 is equal to the change in energy of system after the shift:

$$W = \Delta E = {}^2E_{sys} - {}^1E_{sys} = -E_{12} \qquad So, \qquad E_{12} = -W$$

The ***magnitude*** *of binding energy is equal to the work, one must spend to detach the two (or all, if there are more) constitutive parts of the system and shift them far away from each other. The binding energy is* ***negative*** *because a bound system is a stable configuration (lower energy than the* ***unbounded configuration.e. no system or in other terms zero energy****).*

So, the atomic and nuclear binding energies have negative values. But, do not forget that it is the difference between the energy values that have a physics sense, not the energy values themselves.
Example*: The molecular binding energy of molecule H_2O is equal to the work one must do to send to infinity the three atoms H, H, O.*

-Having the same electric charge, the protons inside the nucleus, push each other. The fact that they remain bounded means that another attractive force with bigger magnitude acts between them.
This is the nuclear force which main characteristics are:

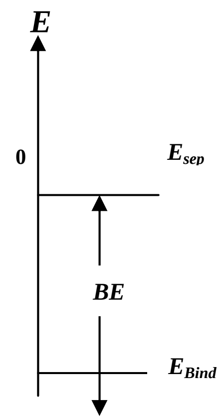
a) Its attractive action fades quickly for distances > 3 - 4fm. It's a short distance force.
b) It does not depend on nucleon electric charge. It's the same for protons and neutrons.

Nuclear forces are (1)<u>attractive</u>, (2)<u>short range</u> forces that (3)<u>do not depend</u> on electric charge.

-The **<u>nuclear binding energy</u>** (**BE**) is the required energy for separation of all particles inside a given nuclide. To calculate BE one may use the step-by step following logic:

a) ***Z*** protons and (***A-Z***) neutrons constitute the nuclide which is a <u>stable system</u>.
b) This means that the ***system of protons and neutrons*** <u>*has smaller*</u> energy when they are inside this nuclide compared to the <u>*sum of their energy*</u> when separated.

When gathered inside the <u>*neutral atom "x" containing Z electrons*</u>, ***the mass*** and the ***energy*** of these nucleons are



$$m_{nucl\_inside\_atom} = m_x - Z*m_e \quad \text{and} \quad E_{nucl} = [m_x - Z*m_e]*c^2 \qquad (3)$$

(m_x-the mass of neutral atom)

When separated (each far away from the others) their total energy is (using $E = m*c^2$)

$$E_{sep} = [Z*m_p+(A-Z)m_n]*c^2 \qquad (4)$$

By adding and subtracting the mass of $Z*m_{e-}$ to the expression (4), it transforms to

$$E_{sep} = [(Z*(m_p+ m_{e-}) + (A-Z)m_n - Z* m_{e-}]*c^2 = [Z*m_H + (A-Z)m_n - Z* m_{e-}]*c^2 \qquad (5)$$

m_H is the mass of neutral hydrogen atom.

Then , the binding energy of nucleons is expressed as

$$BE = E_{sep} - E_{nucl} = [Z*m_H+(A-Z)m_n - Z*m_e - m_x +Z*m_e]*c^2$$

$$BE = [Z*m_H +(A-Z)*m_n - m_x]*c^2 = \Delta m* c^2 \qquad (6)$$

Where the quantity ***$\Delta m = [Z*m_H +(A-Z)*m_n - m_x]$ is known as "nuclear mass default".***

<u>Example</u>: Calculate BE for ${}^{12}_{6}C$ and average BE per nucleon in ${}^{12}_{6}C$.

a) The mass of neutral C atom is m_C = 12 u. We have Z = 6 and A-Z=12- 6 = 6 and m_H = 1.007825u ; m_n= 1.008665u .

So, Δm = [6*1.007825+6*1.008665 - 12]u = 0.09894u ; As 1u = 931.5 Mev/c^2
<u>BE</u> = $\Delta m *c^2$= (0.09894u)*c^2 = (0.09894*931.5Mev/c^2 **)***c^2 = **<u>92.16261Mev</u>**

b) The average energy binding per nucleon <u>BE/A</u> is 92.16261/12 = **<u>7.680217Mev</u>**

NUCLEAR STABILITY

- The <u>*average binding energy per nucleon*</u> (***BE/A***) is a key parameter for understanding the <u>*nuclear stability*</u>. The figure 2 shows that it increases 3 times from deuterium (~1MeV) to tritium (~3MeV) and continues to increase <u>***linearly***</u> until A~30. Then, it increases ***gradually*** till maximum value 8.75 MeV (nucleus ${}^{56}_{26}Fe$) to follow by a decrease to 7.6MeV at the last natural element ${}^{238}_{92}U$. This curve informs about the <u>*spatial range of action for nuclear forces*</u>. As long as the ***average binding energy*** of *nucleon increases with A,* the ***nuclear attraction force*** is bigger than the ***repulsive*** (p-p) ***electric forces***; this happens for A = 1 to 20-30. For A > 30 there is no significant increase of average binding energy. In general terms, this tells that the *attractive action of nuclear forces starts to be comparable to electric repulsion action between protons.* As the *electric force does not change significantly in so short distances it comes out that it is the nuclear force action that decreases*. We conclude that the nuclear forces are ***short-range forces*** acting strongly ***till a maximum distance*** (≈radius of nuclide with A=30)

$$R \leq 1.2 * 30^{1/3} = 3.73 \text{ fm} \qquad (7)$$

- The graph in fig 3 shows the variation of number of neutrons for the natural nuclides (Z = 1- 92). One may see that the light atomic nuclei contain practically as many neutrons as protons.
Above Z = 20 (Ca), the atomic nuclei require a higher number of neutrons, in order to compensate for the increasing electric repulsion protons-proton. As long as the number of protons is not too high, the (attractive) nuclear forces between all nucleons win over the repulsive Coulomb forces between the protons. The atomic nuclei remain stable as long as that they contain an adequate number of neutrons, in order to "dilute" the concentration of positive charges brought about by the protons. With more than 83 (Z= 83) protons, <u>*irrespective of the number of neutrons*</u>, the atomic nucleus is unstable. The nucleus ${}^{209}_{83}Bi$ (bismuth) is the heaviest element of which at least one isotope is stable. The nuclei with **Z > 83** suffer the ***radioactive disintegration***.

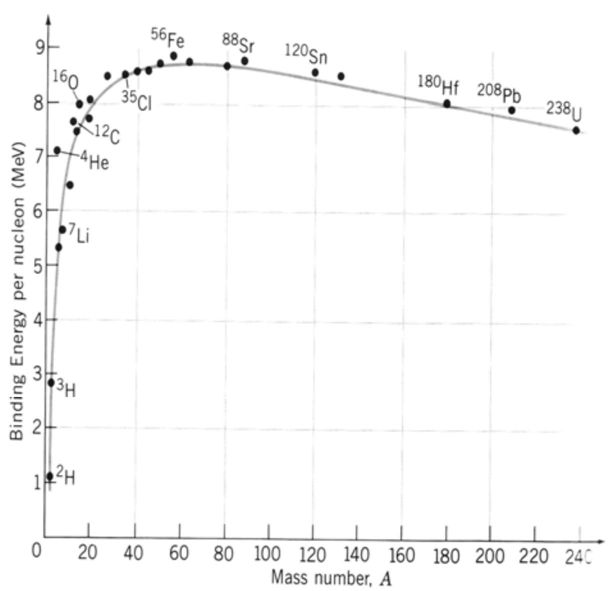


Fig. 2

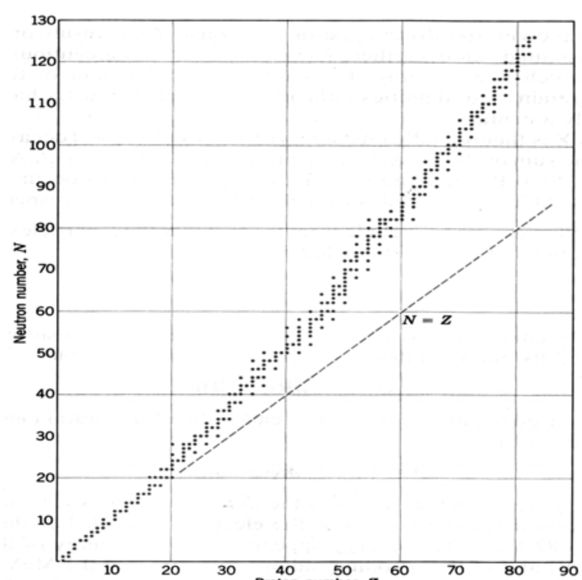


Fig.3