REMEMBER:

-When dealing with one aperture (or obstacle) there is a "cut" on the wave front and this cut is always associated by a "deformation" of wave front at the "output" of aperture. This phenomenon is called diffraction. The presence of a signal (radio, TV, sound, light) inside the "geometrical shadow" and diffraction patterns beyond an illuminated small aperture are *observable results* of diffraction.

- We calculated the diffraction pattern from a slit by use of interference principles. In optics, these two phenomena are closely related. So, to make a difference people call "diffraction pattern" those corresponding to **one** aperture (or obstacle) and interference patterns those corresponding to a system of **several** apertures (or obstacles).

OPTICAL SPECTROSCOPY

-The dispersion of sunlight into a spectrum of different colours by using a glass prism (fig.1) brought to the idea that one may gather information about the source by analysing the emitted light. This was the first step for the development of the optical spectroscopy.

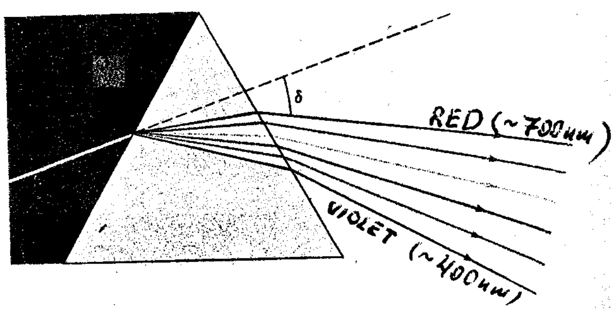


Fig.1

-We will see that, when a light source contains only one element (one type of atoms), the emitted light is constituted by a specific finite set of wavelengths ($\lambda_1, \lambda_2, \lambda_3, \lambda_4, ... \lambda_f$) and this set is a "signature of this element". One can use a spectroscope to see these λ-sets or a spectrometer to **record** them and ten to identify the elements in a sample.

-For along time, the scientist used optical spectrometers based on the dispersion features of prism mounts. These devices have moderate resolution (2-10 nm in visible region) and do not allow distinguishing between wavelengths closer than 1 nm. When studying the interference from a system of equal slits, the scientists realised that these systems could improve essentially the resolution of light spectrometers. Nowadays, optical gratings constitute the central unit for the majority of optical spectrometers.

-The capacity to distinguish between two close wavelengths λ_1, λ_2 is a major characteristic for a spectrometer. The resolution of a spectrometer is higher if the minimum observable **Δλ** is smaller; $R \sim 1/\Delta\lambda$. As **Δλ** depends on the wavelength region the scientists follow the resolution of the spectrometer by use of **the resolving power** .

$$R = \frac{\lambda}{\Delta\lambda} \qquad (1)$$

Around the middle of visible light, the **resolving power** of a prism spectrometer is $R_{prism} = \frac{500nm}{5nm} = 100$ while for a grating it is easily > 1000.

GRATINGS

-All optical grating contain parallel grooved lines onto a glass surface. The space between scratched grooves plays the role of a diffracting slit(known as *ruling* or *lines*). In a transparent grating (fig.2a), the input wave is perpendicular to the grating surface while the output waves travel on the other side of grating. In a reflection grating (fig.2b), the input wave and the output waves are on the same side of the grating; the input wave is perpendicular to the surface and the output waves are inclined. The spacing "***d*** " between centers of adjacent slits is called ***grating spacing***.

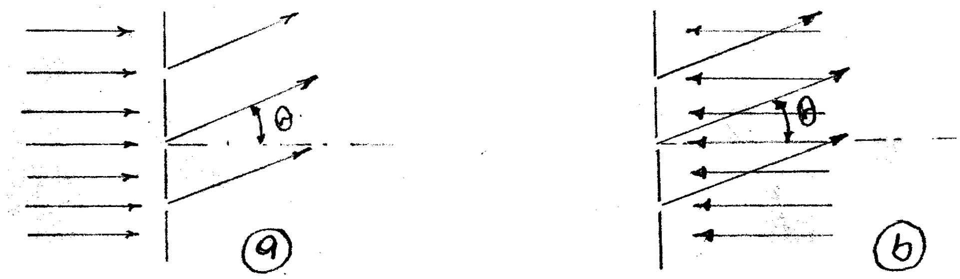


Fig. 2

-An optical grating operates as a dense set of parallel fine slits. The lines are very fine. The central maximum of diffraction of such lines covers the screen. Note that the screen has to be enough distant so that the wave front of one slit is plane.

Example. One grating has 500gr/mm. Estimate the distance one has to locate a screen with height 50cm so that it is illuminated uniformly by a single slit?

Consider a wave at the middle of visible spectrum ($\lambda = 500$nm).

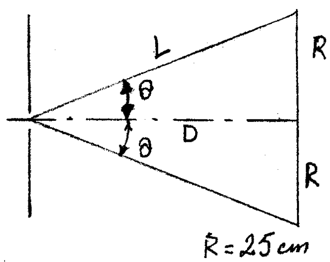


Figure 3

Each slit produces on the screen a diffraction pattern with a central bright maximum. We want the extension of this maximum be enough to cover the screen height. This means that the first minimum must be located at the border of the screen. The first minimum position is defined by equation

$a\sin\vartheta = \lambda \rightarrow \sin\vartheta = \dfrac{\lambda}{a}$ (*) **a** is the " slit aperture".

From our data the distance between two adjacent lines is d =1mm/500 = $2*10^{-3}$ mm= $2*10^{-6}$ m = $2*10^{3}$ nm. Knowing that ***a*** < ***d***, we make a conservative choice[1]; ***a* = *d***. From the figure 3, one may see that

$$\sin\vartheta = \frac{R}{L} = \frac{R}{\sqrt{D^2+R^2}} = \frac{\lambda}{a} = \frac{500}{2000} = 0.25 \qquad \frac{R^2}{D^2+R^2} = (0.25)^2 = 0.0625 \qquad R^2 = 0.0625(D^2+R^2)$$

$$(1-0.0625)R^2 = 0.0625D^2 \rightarrow D = \sqrt{\frac{0.9375}{0.0625}} * R = 3.87 * 25cm = 96cm$$

-So, for groove density around several hundreds/mm or higher, the diffraction pattern of each single slit produces a uniform illumination on the screen. Let's consider that a monochromatic plane wave falls perpendicular on the grating surface. Each slit produces a uniform illumination on the screen. As the waves from different slits super-pose on the screen surface and as they are all coherent it comes out that an interference picture will be produced on the screen. We may define the location of maxima and minima by calculating the path difference between different waves on the screen surface.

[1] As the angle θ_1 ,see eq. (*), increases when a decreases, we are referring to a limiting higher value of a.

-Let's consider a system (fig.4) of "point sources" equally spaced (distance = d) and a monochromatic (one λ) wave at its input. Suppose that the angle θ in figure 4 is such that path difference between rays 1 and 2 is λ. That is

$$\delta_{1-2} = d\sin\vartheta = \lambda \qquad (1)$$

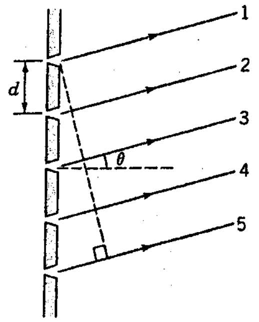


Fig.4

As the corresponding phase difference is

$$\Delta\phi_{1-2} = \frac{2\pi}{\lambda} * \lambda = 2\pi \qquad (2)$$

rays 1 and 2 produce a **maximum** of interference along this direction. One may easily verify that the same conditions 1 and 2 are fulfilled by each couple of **adjacent** rays(3-4,5-6,....). So, the maximum direction for the a slit system is the same as that of two slits. In general, there is a **maximum** each time the path difference between adjacent slits fulfils the condition

$$\delta = d\sin\vartheta_m = m\lambda;\_\_m = 0;\pm 1;\pm 2.... \qquad (3)$$

$$\Delta\phi_{1-2} = \frac{2\pi}{\lambda} * m\lambda = m * 2\pi \qquad (4)$$

These directions define the location of PRINCIPAL MAXIMA produced by the grating on the screen. *Remember; they are the same as the directions for a system of two slits with the same spacing 'd'.*

-Note that:

a) the interference of waves from grating adjacent slits redistributes the light sent by individual diffraction pattern of each slit on the screen. For non-coherent slits' waves there is a uniform illumination on the screen. For coherent slits' waves the interference concentrate the light into the principal maxima locations.

b) the total light of the same colour(same λ) is distributed equally between the corresponding principal maxima of different orders.

-Note that even the interference between distant rays produces maxima along the directions of principal maxima. For example:

a) the path difference between rays 1-3 is $\delta_{1-3} = 2d\sin\vartheta_m = 2 * m\lambda$ (5)

the corresponding phase difference is $\Delta\phi_{1-3} = \frac{2\pi}{\lambda} * 2 * m\lambda = 2m * 2\pi$

b) the path difference between rays 1-4 is $\delta_{1-4} = 3d\sin\vartheta_m = 3 * m\lambda$ (5)

the corresponding phase difference is $\Delta\phi_{1-4} = \frac{2\pi}{\lambda} * 3 * m\lambda = 3m * 2\pi$ (6)

-But, the interference between distant slits changes the light distribution in the region between each two principal maxima. Besides the minimum of two slits patterns, new secondary maxima and minima appear. To precise the ideas, let's consider the region between the central and the first principal maximum defined by the condition

$$\delta_{1-2} = d\sin\vartheta_1 = 1 * \lambda \qquad (7)$$

Following this angle (θ_1), the interference between rays 1- 4 realises a maximum of order 3 because

$$\delta_{1-4} = 3d\sin\vartheta_1 = 3*(d\sin\vartheta_1) = 3*\lambda \qquad (8)$$

This means that, between central and first principal maxima, there are **two** other secondary maxima corresponding to orders m = 1 and 2 surrounded by **3minima**.
The interference between rays 1- 6 realises a maximum of order 5 following the angle θ_1, because

$$\delta_{1-6} = 5d\sin\vartheta_1 = 5*(d\sin\vartheta_1) = 5*\lambda \qquad (9)$$

This means that, between central and first principal maxima, there are **four** other secondary maxima corresponding to orders m = 1, 2, 3, 4 surrounded by **5 minima**.

The figure 5 shows the modification of the region between the principal maxima when the number of equally distant slits increases from 2 to 8. The presence of 7 minima concentrates the light inside principal maxima and makes them sharper.

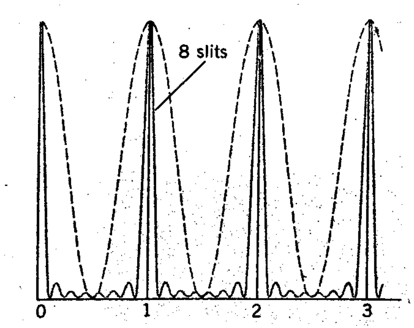


Fig. 5

So, when the number of slits N increases:

a) The principal maxima become narrower and higher;
b) The region between adjacent principal maxima contains secondary small peaks that disappear for very high values of N.
c) The position of principal maxima is the same as that of the double slits interference.

<u>SOME BASIC RESTRICTIONS ON GRATING USE</u>

-Calculations show that the **resolving power** of a grating is

$$R = \frac{\lambda}{\Delta\lambda} = N*m \qquad (10)$$

N-total number of slits and m-the interference order.

Based on expression (10) one would attempt to increase the resolving power by working with high order of interference. But there are two basic restrictions:

a) The order of interference increases with the deviation angle θ but, at first, it has to obey to the principal maxima condition:

$$d\sin\vartheta_m = m\lambda \rightarrow m = \frac{d}{\lambda}\sin\theta_m.$$

As $\sin\theta \le 1$ the **maximum possible** value for the interference **order** is

$$m_{max} = \frac{d}{\lambda} \qquad (11)$$

b) Simple calculations show that for m =3,4... there is an overlap of different order spectra. This situation makes that one uses the principal maxima of second order and avoids the maxima of higher orders.

c) As the number of slits for a general purpose grating is easily N >1000, one may understand that resolving powers of order R = 2000 are very common for grating spectrometers. This is the big advantage of gratings towards prisms. But, a grating that function on its second order has to distribute the falling light of a given λ into five principal maxima. Meanwhile, the prism collects all this light inside one maximum that is located along the direction reserved for that λ by its dispersion. So, in situations when the power of input signal is very low and there is no need for high resolution, one prefers the prism spectrographs.

THE PHASOR

-We have introduced the trigonometric model for SHO study and we have used it:

a) for the derivation of wave function in SHO;

b) to find the relationship between the phase shift and path difference $\phi = \frac{2\pi}{\lambda}\delta$;

We have used this model to find the interference rules for two coherent waves. Also, we used trigonometric calculations for fringes' intensity in Young's experiment and in these calculations we dealt also we two waves. One may think to use the same method (trigonometric calculations) in case of multiple waves but this is not practical. The calculations become cumbersome even for three-wave interference. It is clear that one has to use another method to calculate the patterns' intensity in case of grating where a big numbers of waves interfere.

-The concept of PHASOR is an extension of our initial trigonometric model and offers full flexibility in the studies of interference from multiple waves.

a) It is a vector. As a consequence all vector operations are valid for phasors. It represents a physical quantity that oscillates sinusoidal in time. In case of light, the physical quantity is the electric field of light wave; $E = E_0 \sin(\omega t)$

b) Its magnitude is equal to wave amplitude (E_0 for light waves).

c) At a given moment 't' it forms the angle φ(t) – "phase" with a reference axis "Ox axis" and rotates with constant circular frequency ω.

d) The projection of phasor on to "the vertical axis Oy" represents the variation of physical quantity in time.

e) If several oscillations of the same nature superpose, one may study the result behaviour by use of the sum of respective phasors.

PHASORS IN MULTIPLE SLITS INTERFERENCE

-In the following we perform calculations for a given λ and assume:

a) Actual field vectors lie along the same direction in space.

b) Very narrow slits which, when alone, produce uniform illumination on the screen.

c) Equal separation 'd' between each two adjacent slits.

d) The screen is located in the far-field region. This means that outgoing rays are almost parallel and there is a constant path difference between adjacent slits $\delta = d\sin\theta$.

TWO SLITS SYSTEM

Two phasors with equal magnitude (E_0) rotate all time in phase (same ω).

a) For phase shift φ = 0, 2π,..

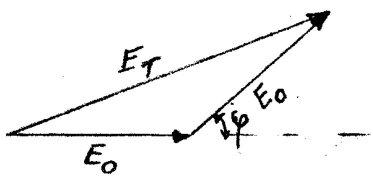
The sum phasor has the magnitude $2E_0$. The corresponding intensity is $4E_0^2 = 4I_0$.

b) For phase shift φ = π, 3π,5 π..

The sum phasor has the magnitude 0. The corresponding intensity is 0.

So, the intensity values vary between the maximum value $4I_0$ and the minimum value 0.

c) For other values of phase difference (see fig.6)



From the theorem of cosines one finds out that

$$E_T^2 = E_0^2 + E_0^2 - 2E_0E_0\cos(\pi - \phi) =$$
$$= 2E_0^2 + 2E_0^2\cos\phi = 2E_0^2(1+\cos\phi) = \qquad (12)$$
$$= 4E_0^2\cos^2(\phi/2)$$

Fig.6

THREE SLITS SYSTEM

Three phasors with equal magnitude and same phase difference φ. The figure 7 presents the resultant phasor magnitude for different φ- values between [0,2π]. That is between the central and first principle maxima.

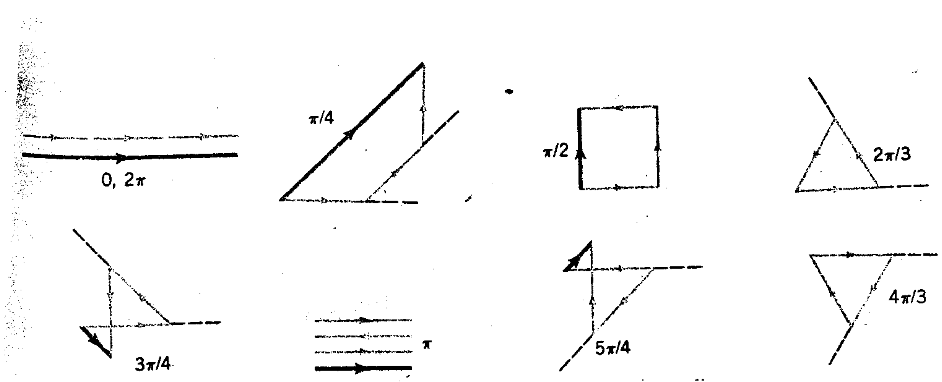


Figure 7

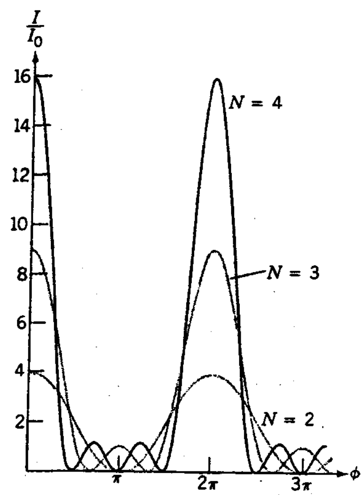
One may get informed about the resultant intensity for different values of phase φ by following the evolution of resultants phasor in figure 7. It is easily seen that it :

a) Is maximum $I_{max} = 9I_0$ for φ = 0 which corresponds to central maxima.
b) Decreases while φ increases to 45^0(π/4) and 90^0 (π/2).
c) Become 0 for φ = 120^0 (2π/3)......... [3*φ = 1*2 π].
d) Increases for φ = 135^0 (3π/4).
e) Get a local (secondary) maximum for φ = 180^0 (π).

f) Decreases for $\varphi = 225^0$ ($5\pi/4$).
g) Become 0 for $\varphi = 240^0$ ($2*2\pi/3$)……… [$3*\varphi = 2*2\pi$].
h) Increases to $I_{max} = 9I_0$ for $\varphi = 2\pi$ which correspond to first principal maxima.

From this scheme we can get the following basic features of three slits interference:

Principal maxima for $\varphi = m*2\pi$; $m = 0, \pm 1, \pm 2, \ldots$
Minima $\varphi = p*2\pi/3$; $p = \pm 1, \pm 4, \pm 5, \ldots$
$p \neq 0, \pm 3, \pm 6, \ldots$
Secondary maxima for $\varphi = (2k+1)\pi$; $k = 0, \pm 1, \pm 2, \ldots$



These features appear in the calculated profiles that are drawn in figure 8. One may calculate these spectra by using N = 2, 3, 4 in the general formulae we will get at the next section.

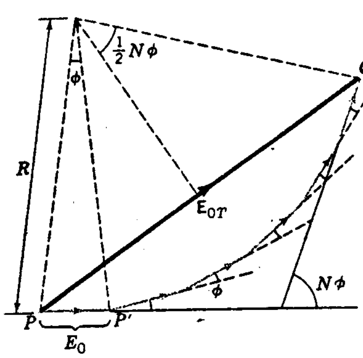
Fig. 8

THE CALCULATION OF PATTERN INTENSITY FOR A SYSTEM OF N SLITS

-The phase shift between two adjacent slits in a system of N equally spaced narrow slits with separation 'd' is

$$\delta = d \sin \vartheta \qquad (13)$$

Figure 9 presents a phasor diagram for an N slits system. Note that one can get quickly the information about the basic features of spectrum.



a) There is a principal maximum for each angle

$$\phi = m*2\pi;\_m = 0, \pm 1, \pm 2, .. \qquad (14)$$

b) There is a minimum for such angles that

$N\phi = p*2\pi;\_p = 0, \pm 1, \pm 2, ..$ or

$$\phi = p*\frac{2\pi}{N};\_p = 0, \pm 1, \pm 2, .. \qquad (15)$$

$$\_\_\_\_\_\_\_\_ p \neq N, 2N, 3N ..$$

c) The first minimum besides the central maximum (important for resolution issues) occurs for

$$N\phi = 2\pi; \rightarrow \phi = \frac{2\pi}{N}\_and\_\frac{2\pi}{\lambda} d \sin \theta = \frac{2\pi}{N}$$

which defines the angle as $\sin \theta^1_{min} = \frac{\lambda}{N*d}$ (16)

Fig. 9

-From expression (14) we see that principal maxima positions do not depend on the number of slits. Their location is the same as that of a two slits system. Meanwhile, the expression (16) shows that they are much narrower in the case of big number of slits.

- Based on the fig. 9 we can find easily the expression for intensity as function of phase difference φ. From this drowing one may see that:

$$\sin\phi/2 = \frac{E_0}{2R} \rightarrow E_0 = 2R\sin\phi/2 \qquad (17)$$

$$\sin N\phi/2 = \frac{E_{0T}}{2R} \Rightarrow E_{0T} = 2R\sin N\phi/2 \qquad (18)$$

Then $$\frac{E_{0T}}{E_0} = \frac{\sin(N\phi/2)}{\sin(\phi/2)} \rightarrow\rightarrow\rightarrow \frac{I}{I_0} = \left(\frac{E_{0T}}{E_0}\right)^2 = \frac{\sin^2(N\phi/2)}{\sin^2(\phi/2)} \qquad (19)$$

The expression (19) is very general. It gives the intensity value for each angle and for each number of slits. For example, for two slits we have

$$I = I_0\frac{\sin^2(2*\phi/2)}{\sin^2(\phi/2)} = I_0\frac{\sin^2(\phi)}{\sin^2(\phi/2)} = I_0\frac{[2\sin(\phi/2)\cos(\phi/2)]^2}{\sin^2(\phi/2)} = 4I_0\cos^2(\phi/2)$$

So, we find the known results for pattern intensity in two slits interference

$$I = 4I_0\cos^2(\phi/2) \qquad (20)$$

N-SOURCES INTERFERENCE IN OTHER REGIONS OF E.M. SPECTRUM

- Note that light waves are electromagnetic waves and we did not do any special restriction on the wavelength of interfering waves. So, the principles we used and the derived conclusions might apply in other regions of E.M. spectrum, too.

- The model of N equally distant source played a decisive role in the discovery of physical nature of x-rays. At the beginning of the last century, the scientists knew that a space dimension ~ 0.1nm was characteristic for these rays but didn't know if they were dealing with a particle or a wave. It was the diffraction of x-rays from the atom arrays in a crystal that proved their wave nature at 1922. As the distance between the atoms is of the same order as x-rays, a system of identical equidistant atoms acts as a system of identical equidistant slits. Nowadays, the x-ray diffraction is the main technique that infers information about the arrangement of atoms in crystal structures.

- The principles of interference from equally distant identical sources are applied to amplify the low intensity signals. This is the case of a system of similar telescopes that receive small signal from one direction in space. All signal are superposed at a center. A special electronic treatment builds the precise phase shift that would correspond to a principal maximum of interference for the λ used by the telescopes.