# COHERENCE AND INTERFERENCE

- At any interference experiment, one must use **coherent waves** **(same frequency and wavelength).** To observe a ***stable pattern*** of interference, it is essential that the **phase shift** ($\Delta\varphi_{tot}$) between the waves that interfere at any <u>*given screen point*</u> **remains constant** during **observation time**. In two slits interference model, we considered two waves $E_1 = E_0 \sin(\omega t)\_and\_E_2 = E_0 \sin(\omega t + \varphi)$ (1)
This modelling is based on three assumptions **a)** *Infinite long waves in time;* **b)** *Same frequency* ;
c) *constant phase shift* all time. Let's verify the limits of our assumptions and make some corrections.

**a)** Experimental measurements show that the *visible wave light* ($\lambda \sim$ 550nm) is <u>*emitted by the atoms*</u> during $\boldsymbol{\tau \sim 10^{-8}}$***sec***. This wave associates the transition of atom from a high to lower *energy level* and it is emitted *randomly in time and space direction.* As the period of this wave is $\boldsymbol{T} = \lambda/c$ we find that $\boldsymbol{T} = \lambda/c = 550*10^{-9}/\ 3*10^{8} = 1.833*10^{-15}$ sec. This means that in the wave train emitted by the atom there are $\tau / T \approx 10^{-8}/1.833*10^{-15} = 5.5*10^{6}$ ***<u>full periods</u>***. During the emission of one period from the source, the wave front advances by one wavelength. So, for $\boldsymbol{\tau \sim 10^{-8}}$***sec*** the source (*atom*) emits $\tau / T = 5.5*10^{6}$ full oscillations in row; i.e. a length of $(\tau / T)*\lambda = 5.5*10^{6}*550*10^{-9} = 3m$. So, the *average length* of *randomly emitted visible wavelet* (fig 1.a) by a single atom is $\boldsymbol{L_c \sim 3m}$. This parameter is known as the wave ***COHERENCE LENGTH*** . Actually, *due to collisions with other atoms*, the real coherence length is much smaller ($L_c \sim$ *20-30cm*) even for the best conventional light sources (*low density Cs gases*). It is almost the same for common He-Ne but can be as long as 3km at some special lasers. So, <u>the spatial length or coherence length</u> of *interfering waves* ***is not infinity***.

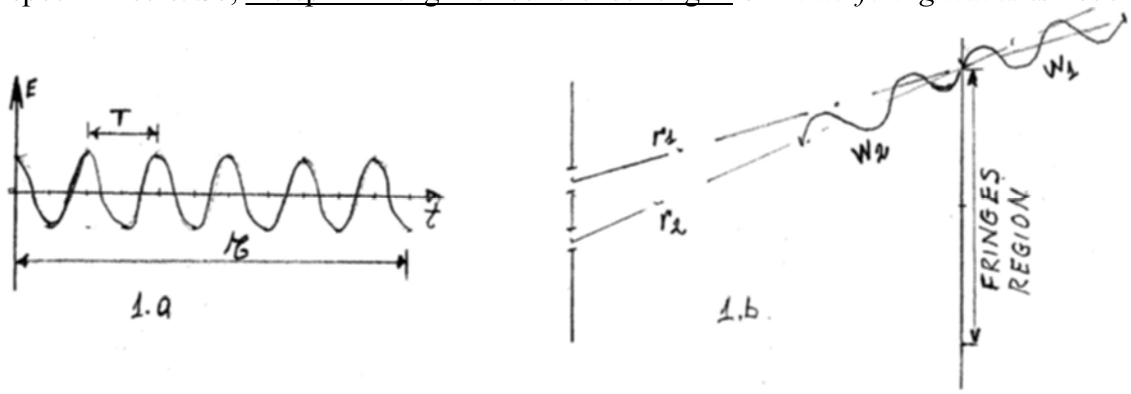



Fig 1

This length ($\mathbf{L_c}$ ) restricts the extension of region occupied by fringes in an interference experiment. The interference calculations are based on "path difference ($\boldsymbol{\delta = r_1 - r_2}$ ). But, at first, the two ***wavelets*** have to superpose and if $\boldsymbol{\delta > L_c}$ they cannot (fig 1.b). This is an effect that restricts the interference order of fringes to a certain *maximum* ***order of interference*** $\boldsymbol{M_{max}}$.

**b)** ***Same frequency*** is a precise modeling for ***two atoms*** of the ***same element***.

**c)** ***Constant phase*** shift all time is ***impossible*** because the ***process*** of irradiation is ***random***. At Young's experiment, this problem is overcome by using a pinhole that selects a few secondary wavelets emitted by a restricted zone of <u>*initial wavefront*</u> and *two* slits that get two "*in phase oscillations from the same* ***<u>plane wavefront</u>***" due to a considerable distance from the pinhole. The phase of the wave front that hits the two slits changes randomly in time but, *at a given moment, it is the same at all points across the two slits*. So, although the **phase at the "entrance" of slits oscillates randomly, the difference of phases for the two outgoing wavelets is equal to zero , all time**.

- In the process of light emission participates a big number of atoms and the light wave produced by the source is constituted by a big number of wavelets. The average length $\mathbf{L_c}$ of those wavelets is larger if the random collisions between the atoms are minimized; a low pressure gaz source does have advantages from this point of view. The production of a big number of wavelets in phase inside the source means a certain intensity of package of the emitted wavelet by the source. This requires the simultaneous emission by many atoms inside the source.
  These two interference requirements are met by the laser light where the <u>random effect</u> *of wavelets emission* from different atoms in the light source is minimized. This makes possible the excellent coherence proprieties of laser light. One recognizes two kind of coherence (fig 2) :

  a) ***Temporal coherence*** = **is** increased if the emission process for each atom is not random. This means large values of ***Lc* ( *coherence length* or *wavelet length)***

  b) ***Spatial coherence*** = all atoms emit wavelets ***simultaneously*** (**same phase**) and even along the ***same direction in space***.

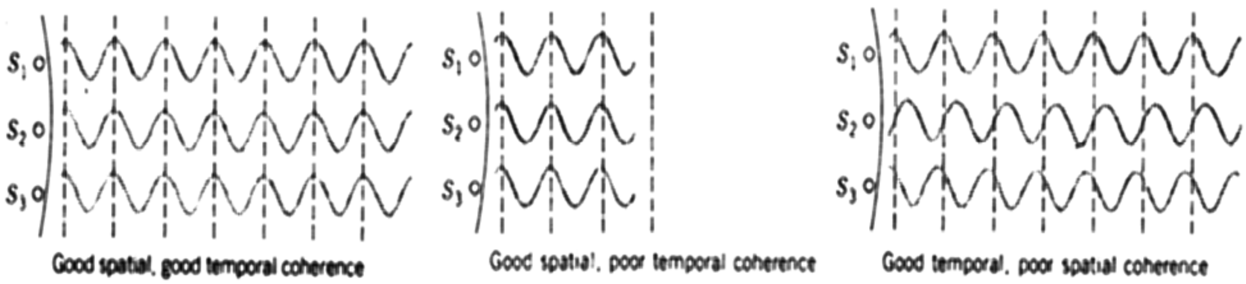



Fig2 ***(Short Lc)*** ***(Longt Lc)***

- When calculating the interference from two waves (1) we found the resultant intensity is

$$I = 4I_0 \cos^2(\Delta\phi/2)\_remember\_I_0 = E_0^2 \qquad (2)$$

Note that this result and its graph in figure (3) assume a constant phase shift *Δφ* between interfering waves. For a random *Δφ* (each slit illuminated by a different source) one has to work with *Δφ* -average.

As for each point on the screen, *Δφ* changes in [0,2π] randomly, one finds out $\overline{\cos^2(\Delta\phi/2)} = 1/2$ and

$$I = 2I_0 = I_0 + I_0 \qquad (3)$$

This corresponds to the uniform illumination in case of the superposition of two completely incoherent waves. Remember that the average light intensity on the screen for coherent and non-coherent waves is the same, $2I_0$ (see fig 3). But, the interference redistributes it differently on the screen.

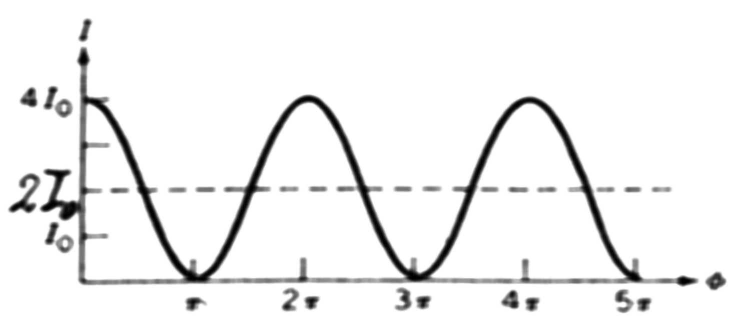



Fig 3

## LIGHT DIFFRACTION

-In general, the term diffraction is applied to situations involving the resultant effect produced by a limited portion of wave front. In optics, it appears as " ***light bending around obstacles***" and produces some light in regions where geometrical optics predicts shadow.
Note that ***diffraction effects*** are observed always when a *part of wave front is cut*. As all optical devices use a limited portion of wave front, they cannot avoid diffraction.

-The figure 4 shows a diffraction pattern formed by a circular obstacle on a screen at a moderate respective distance. There are alternating dark /bright fringes around the image borders and a central bright spot (Poisson spot). This situation corresponds to the "FRESNEL DIFFRACTION": *Either the light source or the screen is close to the obstacle (or aperture).* In these circumstances, the **wave fronts are spherical** (*not planes*) and the calculations go beyond the limits of this course.

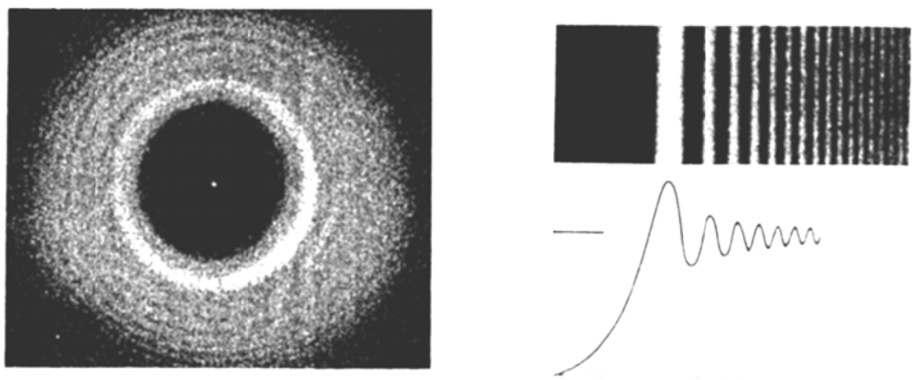


Fig.4

-We will study the diffraction pattern when *the source and the screen are **far** from the aperture (or obstacle). In this case, the incident light advances by **plane fronts of wave** and parallel rays strike the screen*[1]. This situation is known as FRAUNHOFER DIFRACTION.

## SINGLE SLIT FRAUNHOFER DIFFRACTION

-This is the situation for *each of slits* used in Young's experiment.
We will see the effect of ***aperture dimension*** on *far field interference* for a ***monochromatic* (one λ) *plane wave front*** at slits' input.

-Consider the division of slit aperture "**a"** into twelve thin strips of equal infinity small thickness and parallel to the slit. Each strip element (**presented by a point in fig.5**) emits one **Huygens wavelet** of light. All those wavelets are **coherent to each other** and have the same coherence length ($L_C$). It is clear that for the ***direction*** $\theta = 0^0$, all those wavelets interfere ***in phase*** to each other and produce a **maximum of interference** because the path difference between each of them is zero and ***all phasors are aligned*** along the same direction (fig.6.a).

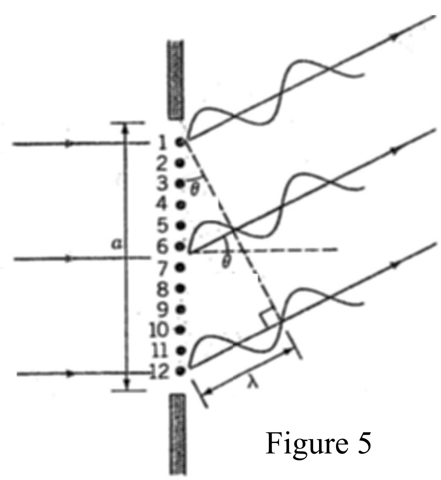



Figure 5

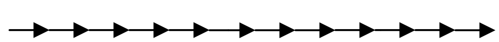


Figure 6.a Twelve phasors "***in phase***"

[1] In optical devices, those rays pass through a **convergent lens** before hitting the **screen located at lens focal plan**.

- For angles $\theta \neq 0^0$ a path difference is produced between the wavelets. Consider the ***angle θ for which the path difference*** between the ***first*** and the ***second wavelets*** is equal to **λ/12**(see fig. 5). In this case the *path difference* between wavelets emitted by the *strips one and twelve* is $\boldsymbol{\delta_{1-12}=\frac{11}{12}\lambda}$. This means that the *phasor* ***twelve*** is phase-advanced by $\Delta\varphi_{1\text{-}12} = k^* \delta_{1\text{-}12} = (2\pi/\lambda)^* \frac{11}{12}\lambda = 2\pi^* \frac{11}{12}$ with respect to phasor **one** and the sum of all phasors gives a <u>zero net phasor </u>(see fig. 6.b). To increase the *precision* of this calculation we must increase the number of stripes and decrease their thickness. Noting that, for n strips the path difference between the first and the last wavelet is

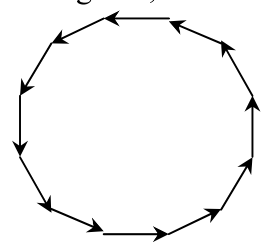


Fig. 6.b

$\delta_{1-n} = \frac{n-1}{n}\lambda$ , it comes out that the corresponding phase shift is

$\Delta\phi_{1-n} = \frac{2\pi}{\lambda} * \frac{n-1}{n}\lambda = \frac{n-1}{n} * 2\pi.$ When the number of stripes becomes very big $\lim_{n\to\infty}\frac{n-1}{n} = 1$ , the **path difference** becomes $\delta_{1-n} = \lambda$ and the **phase shift** becomes $\Delta\phi_{1-n} = 2\pi$

-This analysis shows that along this **direction θ** for which the path difference between the first and last wavelets is $\delta = a\sin\theta = \lambda$ the **net phasor is zero** and consequently **a minimum** of interference is produced. A step by step mathematical procedure based on the phasors' modeling shows that there is a ***minimum*** for each direction that corresponds to the general condition[2]

that
$$a * \sin\theta_s = s\lambda;\_ s = \pm1, \pm2, \pm3, ...... \qquad (4)$$

Note that:

**a)** **s = 0 is excluded** in equation (4) because it corresponds to the ***central maximum*** which is *large* and becomes *larger* with *decrease of slit thickness* (a). (Why?)

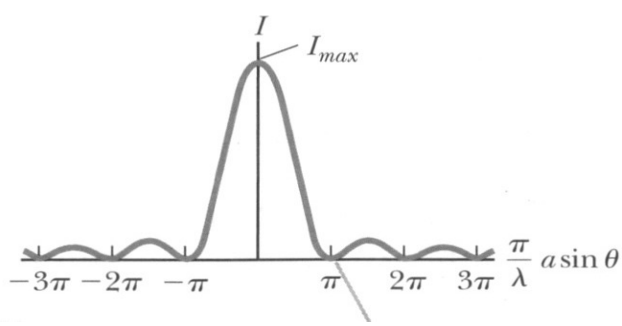
**b)** As "*a*" is small, ***sinθ (= λ/a) may be out of "small angle approx" range even for s =1.***

**c)** In between two consecutive minima there is a maximum. The directions for single slit ***secondary*** **maxima** fulfil the condition

$$a * \sin\theta_S = (2S+1)\lambda/2;\_ S = \pm1, \pm2, \pm3, ...... \qquad (5)$$

**d)** The intensity of 1-slit ***<u>diffraction pattern</u>*** decreases quickly with ***distance from screen center*** and is described by the following function ($\mathbf{I_{max}}$ stands for the *top intensity* of *central maximum*)

$$I = I_{\max}\left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\vartheta/\lambda}\right]^2 \qquad (6)$$




-The formulas (5, 6) tell that even a single slit produces ***maxima-minima patterns*** in the conditions of Fraunhofer diffraction. Similar calculations show that small obstruction (hair, blood cells, ..) produce ***diffraction related*** patterns, too. One uses these patterns to estimate "small objects" dimensions. These ***diffraction related patterns*** are visible only when **a ≥ λ**. If **a** >> **λ** the angles $\theta_s$ are so small that fringes superpose to each other (i.e. disappear) and the image on the screen takes the slit's profile.

[2] The way how to get this formula is given in the textbook, p.791.

CORRECTIONS TO THE RESULTS OF YOUNG'S EXPERIMENT

-When considering the interference conditions for Young's experiment we did not account for the ***diffraction effect of each slit*** pattern individually. Remember that we found:

**a)** the maxima for condition $d * \sin\theta_M = M\lambda \_ m = 0,\pm1,\pm2,\pm3,......$ (7)

**b)** the minima for condition $d * \sin\theta_m = (2m+1)\lambda/2 \_ m = 0,\pm1,\pm2,\pm3,......$ (8)

**Note:** Do not mix **slits' distance 'd' in (7,8)** with **aperture 'a'** in equations **(4,5,6).**

-Suppose that for ***the same direction θ*** the conditions (4) and (7) are fulfilled simultaneously for two slits with equal aperture '***a***' at a distance '***d***'. The condition (5) tells us that in this direction *each slit* produces a *dark fringe*; i.e. *no one slit is sending a light wave at this location*. As there are **not waves to superpose, the corresponding maximum fringe for two slits system will be missing**.

-Suppose that for the same direction θ the conditions (5) and (8) are fulfilled simultaneously for the two slits' system. The eq. (5) tells us that in this direction each slit alone would produce a bright fringe. But the eq. (8) fixes a minimum for this direction and the light brought here by each of slits along this direction is removed. The ***two slits' interference effect "takes this light out of this direction***"(fig. 6).

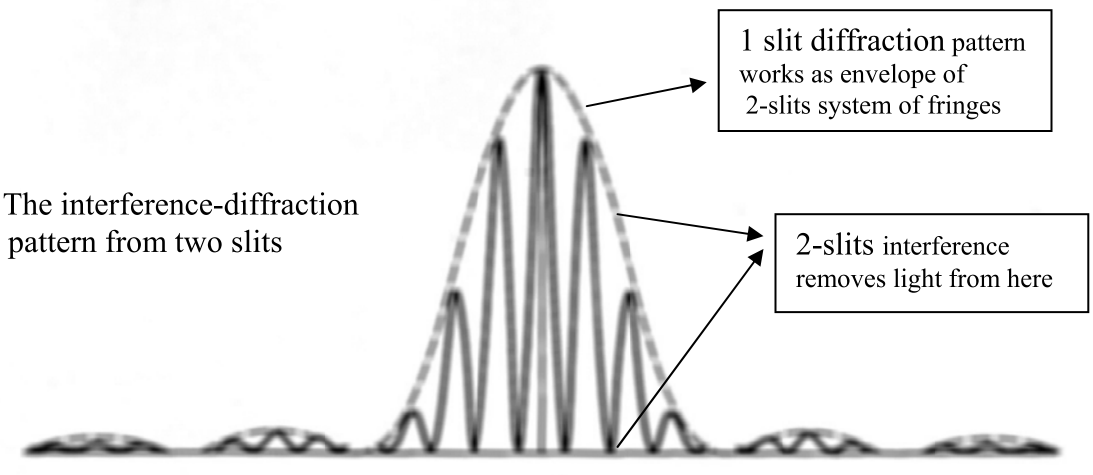



Fig.6 The interference-diffraction pattern from two slits

As the same light must obey simultaneously to two different interference rules it will be a minimum along each direction θ for which exists one minima condition. "It is like **multiplying by zero rule** ". So, several maxima from two-slits interference are cancelled due to one slit diffraction mechanism. The intensities of two slits' maxima pattern are modified in conformity to one-slit diffraction pattern, too.

**Notes**:

a) In general, the distance ***"d"*** between two slits is ***bigger*** than the slit thickness ***"a"***. It comes out that **λ/d << λ/a.** This means that ***several maxima of "2 slit pattern interference" fall between the two first order minima (s = +/-1) due to "1 slit diffraction pattern".***

b) In general, the ***small angle approximation*** does work for two slits interference but, it does not work always for one slit diffraction pattern. So, it is ***useful*** that, before using the small angle approximation, one verify whether the conditions for small angle approximation are met.

## THE RAYLEIGH CRITERION FOR OPTICAL RESULTION

-The ultimate sharpness of a camera images is defined by the **spatial resolution = the capacity to discern two neighbouring points on the object surface.** This parameter is limited by the diffraction.

-In all optical system there is an *input aperture* that allows (cut) only a portion of wave front to pass through lenses. This process is associated with light diffraction. In other words, this means that *even a 'point source' at system input will produce a Fraunhoffer*[3] *pattern* at recording (sensor or film) plane.

-In figure 7, two points on object surface are acting as *non-coherent point sources*. Each of these points produces a similar diffraction pattern on sensor plane but there is ***no interference*** between them to redistribute the light. It can be proved that for a circular aperture with diameter '**a**', the first minimum direction is defined by equation (*proportionality coefficient changes from 1 to 1.22*):

$$\sin\theta = \frac{1.22\lambda}{a} \tag{9}$$

In all optical devices **a >> λ**. So, we use small angle approximation and the equation (9) becomes

$$\theta = \frac{1.22\lambda}{a} \tag{10}$$

This expression gives the *direction of first minimum* with respect with to direction of the center of diffraction pattern for a point source. Taking into account that a similar diffraction pattern is produced by each of two point sources, one gets the picture for intensity distribution shown in figure 7.

Two ***non-coherent*** point sources can be easily resolved as far as their diffraction patterns do not overlap.

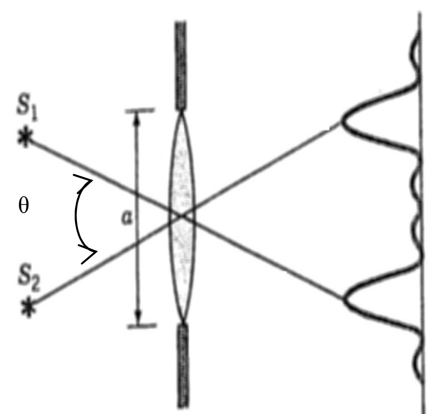



Fig. 7

-When the separation between the two points on the object surface is small, the corresponding diffraction patterns on sensor plane overlap and do not permit to record them clearly as distinct points.

*How to judge whether the images are separated or not on the sensor surface?*

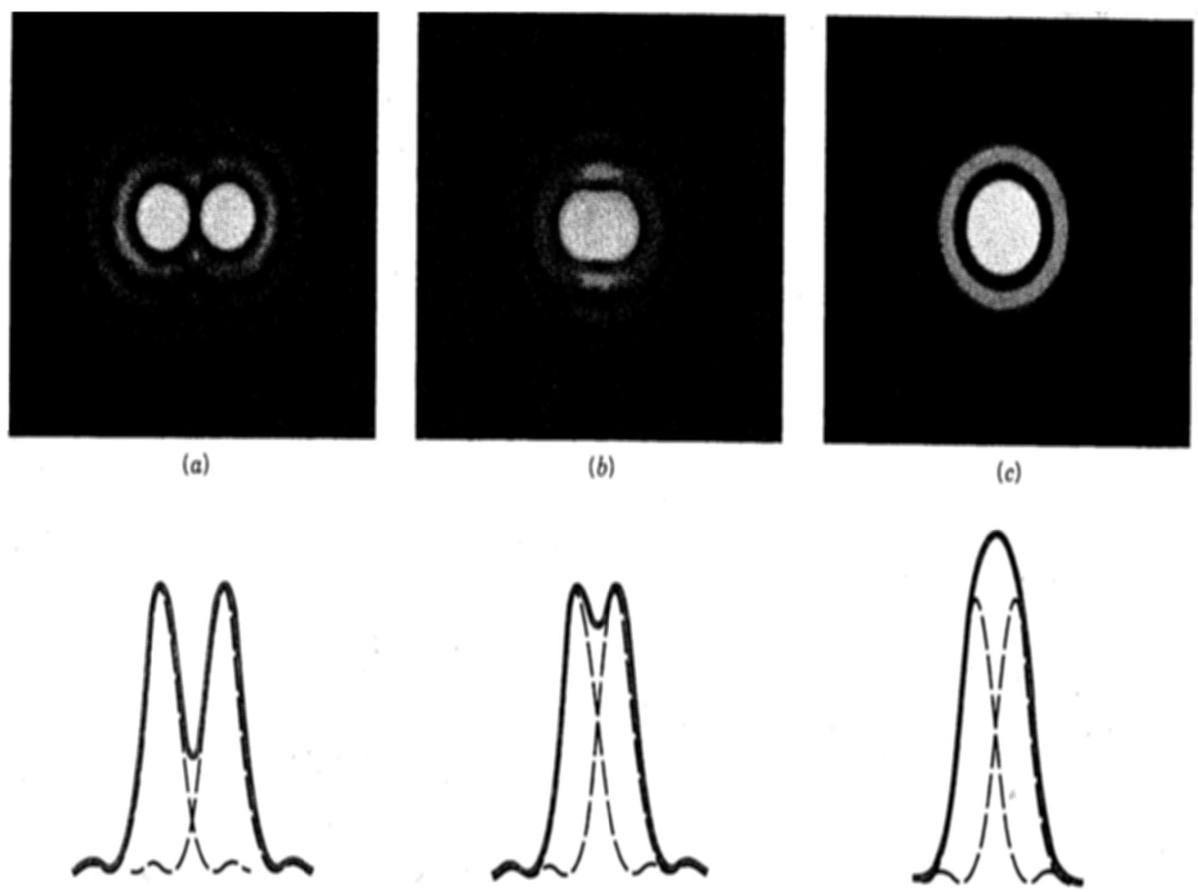
**RAYLEIGH CRITERION**: **Two images are *barely resolved* when the central maximum of one pattern coincides with the first minimum of the other one (see figure 8)**. Following this criterion, the ***critical*** *separation* ***angle*** $\theta_c$ is

[3] We consider a plane front wave at system input and recording plane. Lens brings the image form infinity to focal distance.

$$\theta_c = \frac{1.22\lambda}{a} \qquad (11)$$

For smaller *angular separation* between two objects ($\boldsymbol{\theta < \theta_C}$), the optical system will not distinguish between the two images. One can ***increase resolution*** by using lenses with ***bigger apertures***[4] (the increase of **a** produces a decrease of $\boldsymbol{\theta_c}$ ). This way, one fulfills easier the condition $\theta > \theta_C$ for a *given $\theta$ value* (fixed value) between the set of rays from <u>*two given object points*</u> at lens input (see fig. 7) . Example 38.3 in textbook.

Fig. 8




(*a*) The diffraction pattern of a point source and a circular aperture. (*b*) According to Rayleigh's criterion, two sources can just be resolved if the central maximum of one diffraction pattern coincides with the first minimum of the other. (*c*) If the separation between the sources is reduced further, they can no longer be resolved.

[4] Photographs use this method to improve the quality of recorded pictures.