REMEMBER:

1. Conditions for interference: a) ***same frequency*** (coherent sources);
b) same "displacement" direction for vector fields;

Note: The same amplitude values produce contrasted fringes. Meanwhile, the interference fringes are produced even for different amplitudes but in this case their contrast is low and they may be invisibles.

2. The interference "status" at *a given point* in space is defined from the "**total phase shift**"

$$\Delta\Phi_{tot} = \Delta\varphi_s + \Delta\varphi_r + \Delta\varphi_p$$

$\Delta\varphi_s$ is the phase shift ***between the sources*** of waves that interfere
$\Delta\varphi_r$ is the phase shift due to reflections on each of waves that interfere
$\Delta\varphi_p$ is the phase shift due the "**optical path difference**" between the two waves *at this point*

$$\Delta\varphi_p = \frac{2\pi}{\lambda}\delta_{op} \qquad \delta = n(r_2 - r_1);$$

n is the refractive coefficient of the medium where the waves propagate
r_2 is the distance of the considered point from source 2 ;
r_1 is the distance of the considered point from source 1;

There is an:
interference maximum if $\Delta\Phi_{tot} = +/- M*2\pi$) i.e. if $\delta_{op} = M\lambda \_ M = 0, \pm1, \pm2, ..;$
interference minimum if $\Delta\Phi_{tot} = +/- (2m+1)*\pi$ i.e. if $\delta_{op} = (2m+1)\frac{\lambda}{2} \_ m = 0, \pm1, \pm2, ..;$

3. For all wave phenomena the *diffraction appears* when the *obstacle dimension is ~ λ*.

4. At Young's experiment the positions of fringes are defined by angles as follows

for ***maxima*** $\sin\theta_{\max} = M\frac{\lambda}{d}$; for ***minima*** $\sin\theta_{\min} = (2m+1)\frac{\lambda}{2d}$ M, m = 0,±1,±2,..

Note: The production of fringes at ***Young's experiment proves the wave nature of light***.

5. In cases when we are interested only for relative values of intensity (interference situations), the relation $I = A^2$ is simple and good enough to get out conclusions.
For light waves, $A = E_0$ means that the amplitude of wave "displacement" is the maximum value of the electric component of E.M. wave that describes the light. So, we have for light wave intensity $I_0 = E_0^2$ and the expression for the relative intensity for Young's experiment fringes comes out as

$$I = 4I_0\cos^2(\Delta\phi/2) \Rightarrow\Rightarrow \frac{I}{I_0} = 4\cos^2(\Delta\phi/2)$$

Remember that in two slits' diffraction-interference (Young's) experiment:
- there is maximum if $\delta = M\lambda \_ because \_ \Delta\varphi = (2\pi/\lambda)\delta = M*2\pi$
- there is minimum if $\delta = (2m+1)\lambda/2 \_ because \_ \Delta\varphi = (2\pi/\lambda)\delta = (2m+1)\pi$

THIN FILMS

- Coloured fringes appear often in reflected light from thin films of transparent materials like soap bubbles, oil patches on water or road surfaces,.. These fringes are seen in the reflected light and are produced by the interference between rays reflected at the two borders of thin films.
- Before starting with the explanation of these phenomena it is useful to remember that when a light wave falls from a medium with lower refraction coefficient onto a medium with higher refraction coefficient, a phase change by π[1] is produced in the reflected light and there is no phase change when the light is reflected onto a medium with lower n.
- Let's consider a homogeneous and uniform thickness film limited by media with lower 'n' on both sides (oil in air, soap in air,..). When a light ray falls on the upper boundary, in general, there is a series of reflected and refracted rays that are born(fig. 1)

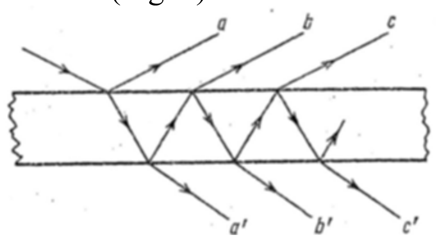

Figure 1

- Note that the ***reflection coefficient*** (*R*= $I_{reflected}/I_{incident}$) of no silvered surfaces is very low. A quick calculation based on R = 0.05 gives the values of table No.1 for the intensities of rays, a,b,c,a',b',c'. ($I_{reflected} = R*I_{incident}$ and $I_{refracted} = (1-R)*I_{incident}$)

Table No1

$I_{reflected}$	Ia = 0.05Io	Ib = 0.045Io	Ic = 0.001Io
$I_{refracted}$	Ia' = 0.9Io	Ib = 0.002Io	Ic = 0.0000001Io

Remember that for an observable interference we need almost the same values of intensity for two interfering waves. Based on this criterion and the data in table 1, we conclude that ***only interference between rays 'a' and 'b' may produce observable fringes.***
So, we follow the study based on only these two rays.

-The interference built by '**a**' and '**b**' rays is defined from their optical path difference plus the additional phase shifts due to reflections. **Remember**: *If the geometrical path of a ray inside the medium with refraction coefficient "**n**" is "**d**", its **optical path** is "**nd**"*. When two rays pass through a medium with refraction coefficient '**n**', their ***path difference δ*** produces an *optical path difference*

$$\delta_{opt.} = n*\delta .$$

When calculating the phase difference in a medium (n ≠1) one has to refer to δ_{opt} and not to δ.
From the figure 2 one may see that the ***optical path difference*** between two first reflected rays (a,b) is

$$\delta_{opt.} = n(AB + BC) - AD \qquad (1)$$

If the first medium is air (***n = 1***) and the incidence angle is θ_1, the refraction angle θ is

$$\sin\theta = (1/n) * \sin\theta_1 \qquad (2)$$

$$AE = AB \sin\theta \; ; \; AC = 2AE = 2AB \sin\theta \qquad (3)$$

[1] From now on, in these cases we will take a positive shift + π.

$$AD = AC \sin\theta_1 = 2AB \sin\theta \sin\theta_1 = 2nAB \sin^2\theta \qquad (4)$$

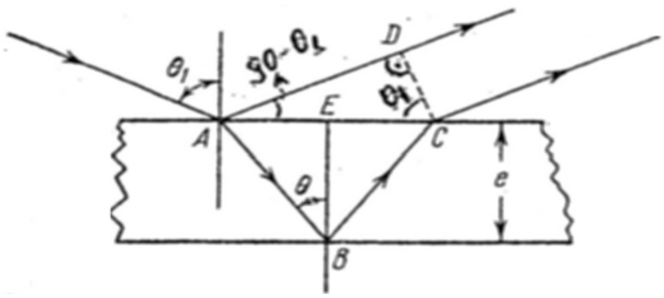

Fig.2

As AB =BC and the total optical length of ray b inside thin film is

$$n(AB + BC) = 2nAB \qquad (5)$$

and we find that the **optical path difference** between the two rays is

$$\delta_{opt} = n(AB + BC) – AD = 2nAB - 2nAB \sin^2\theta = 2nAB(1- \sin^2\theta)= 2nAB\cos^2\theta \qquad (6)$$

From figure 2 is easily seen that $\cos\theta = 1/AB$ (7)

so we get **$\delta opt = 2n*l*\cos\theta$** (8)

- The corresponding phase difference due to this ***optical path difference*** is

$$\Delta\varphi = \frac{2\pi}{\lambda}\delta_{opt} = \frac{2\pi}{\lambda}(2nl\cos\theta) \qquad (9)$$

If we subtract the phase change (+ π) due to reflection at the top interface we find

$$\Delta\varphi_{tot} = \frac{2\pi}{\lambda}\delta_{opt} = \frac{2\pi}{\lambda}(2nl\cos\theta) - \pi \qquad (10)$$

Note that beyond the line DC the phase difference between the two rays remains unchanged.

Important remark: ***One may see the interference produced by these two rays only if they come together inside the iris (3-5mm).*** So, the distance DC must be < 5mm; smaller it is easier the interference is observed. From fig.2 it is clear that DC becomes smaller for incident ***rays close to the normal ($\theta_1 \approx 0$)***. That is why the thin films interference is observed generally close to the normal on films' surface. In this case, $\theta_1 \approx 0$ and as $\theta \approx \theta_1/n$, $\cos\theta \approx 1$. So, the ***optical path difference*** is

$$\delta opt = 2n*l \qquad (11)$$

and the **total phase difference** is

$$\Delta\varphi_{tot} = \frac{2\pi}{\lambda}\delta_{opt} = \frac{2\pi}{\lambda}2nl - \pi \qquad (12)$$

-The other factor that decides on the distance DC and fringes observation is the ***film thickness 'l'***; it must be small (see fig 2). In general, this kind of interference is seen only for ***film thickness equal to several wavelengths***. For films of bigger thickness one must use optical systems to bring rays together inside the iris.

-Let's consider now that we illuminate a thin film by a monochromatic light (one λ) and we observe the reflected light close to the normal to films' surface. There is a maximum

if $\Delta\varphi_{tot} = \frac{2\pi}{\lambda}\delta_{opt} - \pi = 2m\pi \_or\_ \frac{2\pi}{\lambda}\delta_{opt} = (2m+1)\pi \to\to \delta_{opt} = (2m+1)\lambda/2$

So, if the used λ fulfills the condition $\delta_{opt} = 2nl = (2m+1)\lambda/2$ (13) we see a bright light because a **maximum** is produced in the reflected light.

If the used λ fulfills the condition

$\Delta\varphi_{tot} = \frac{2\pi}{\lambda}\delta_{opt} - \pi = (2m+1)\pi \_or\_ \frac{2\pi}{\lambda}\delta_{opt} = 2m\pi \to\to \delta_{opt} = 2nl = m\lambda$ (14) we do not see light because there is a **minimum** in reflected light.

Remember: **a) There is a maximum if** $2nl = (2m+1)\lambda/2$ m = 0, 1,2,3..

b) There is a minimum if $2nl = m\lambda$ m = 0,1,2,3,…

-If we illuminate the film by a white light with equal intensity for each color (simplified model) the condition (13) is fulfilled for several colours and it will be an increase of their intensity in reflected light while the several other colours will fulfill the condition (14) and there will have no reflected light for these colours. As a result, the relative intensities of different colours are pretty much modified in reflected light. This explains why we see coloured light from thin films although the incident one is white (sun light). *The seen colours are the complementary ones of colours missing in reflected light.*

-Here is the place to mention that the complementary colour of a ***primary colour*** (***red, green, blue***) is the colour you get by mixing the other two ;

Missing	***Mixture***	***Seen Colour*** (complementary)
Red	blue + green	cyan
Green	red + blue	purple
Blue	red +green	yellow

LENS COATING

-The reflection coefficient for normal incidence on a surface is calculated as

$$R_{\perp} = \left(\frac{(n_2/n_1) - 1}{(n_2/n_1) + 1} \right)^2 \qquad (15)$$

n_1, n_2 are the refraction coefficients for the first and second medium.

So, for normal incidence of light from air (***n_1=1***) to glass (***n_2=1.5***) or vice-versa, one finds $R_{\perp} = 0.04$.

-When designing an optical system, the output light maximization is one of the main problems because it defines the sensibility of the system. The designer may realize easily that a big quantity of light gathered at input is lost due to reflections onto the surfaces of lens that constitute the system. **Example**: A camera with 5 lenses has 10 reflecting surfaces. Only 66% of the light gathered at camera input [I_{trans}= $(1-0.04)^{10} = 0.96^{10} = 0.66$] is used by the system.

-The solution of this problem is found by coating lens surfaces with such thin films that builds up ***destructive interference*** *on the* ***reflected light***. It is important to understand that a destructive interference for a wavelength in the reflected light is associated with a maximum transmission for this wavelength in transmitted light because the ***interference does not annihilate the energy***.

-In practice, this is achieved by building a thin film that realizes ***equal*** values of ***intensity*** for the ***two first reflected rays*** (*a, b*). It is found that its refraction coefficient has to fulfil the relation (Homework- Hint: Use (15) and look for equal ***Ia*** and ***Ib***)

$$n_{Film} = \sqrt{n_{air} * n_{glass}} = \sqrt{1.5} = 1.225 \qquad (16)$$

As a matter of fact, the lens designers use ***MgF_2*** (***n = 1.38***) because of its high durability that is another important requirement in relation to thin film coatings. The ***coatings thickness*** is selected such that an ***interference minimum*** corresponds to the middle of visible spectrum (λ = 550nm, green range). When a white light falls upon this film, the green colour (λ = 550nm) will be ***missing in reflected light*** and the complementary colour appears on the whole lens surface in reflection. The purple colour (green missing) of cameras' lens is due to this effect. (Homework, Ex. 37.3)

FRINGES OF EQUAL THICKNESS

-In case of a homogeneous and uniform thickness film (***lens coatings***), the condition of ***minima in reflected light*** depends only from λ-value (*one apparent colour on reflection from lens*).
If the film *thickness* is *not uniform*, the apparent colour of reflected light will depend on thickness, too. At positions with *thickness* l_1 the minima in reflection is fulfilled for the wavelength λ_1 and one sees a fringe of apparent colour '*complementary_1*'while at close location with *thickness* l_2 the minima in reflection is fulfilled for wavelength λ_2 and one sees one fringe of apparent colour '*complementary_2'* and so on. This situation explains the ***multiple apparent colours*** seen on ***soap*** and ***oil thin films***.

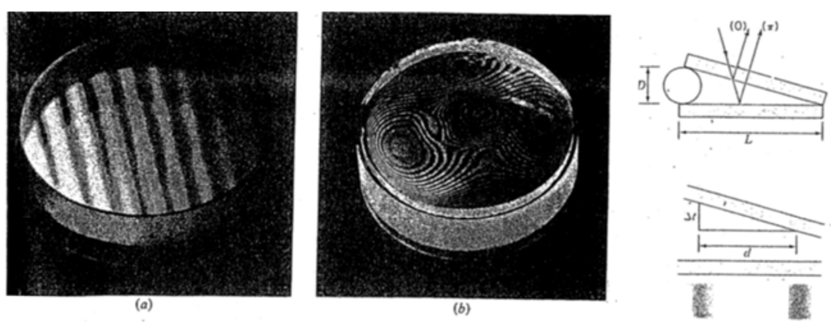

Fig.3

-Thin films effects are used to *control the flatness of transparent plates*. At first, one builds up a *wedge-shaped air film* between the two plates by inserting a fine wire at one end. Then one illuminates them by a monochromatic light (ex. Na lamp). A very clear system of fringes (fig 3.a) parallel to the wire direction appears in reflected light. One may understand easily that the condition for a given maximum (or minimum) is fulfilled at the points over the same thickness of air film and these points are aligned parallel to wire direction.
Note: *Pay attention to phase change due to reflections at film borders. It depends on the value of refraction coefficients for the materials that form the thin film.*

-If one of surfaces is not flat the thickness changes in an irregular way and fringes are not any more parallels to the wire direction. By using this criterion and a reference etalon flat plate, one may control the flatness of other plates during a production process.
(Homework example 37.4)

NEWTONS' RINGS

-A particular case of ***air thin films with thickness variation*** is built by placing a plane - convex lens over a flat plate. In this situation the equal film thickness corresponds to the points on a circle centered to the contact point. When illuminating this system by a monochromatic light, one may see (*with a low power microscope*) circular fringes in the reflected light. At the **center** of fringe system there is a **dark spot** (fig4).

Newton who was the first to observe these fringes tried for a long time to explain the origin of the dark spot but he was unsuccessful.

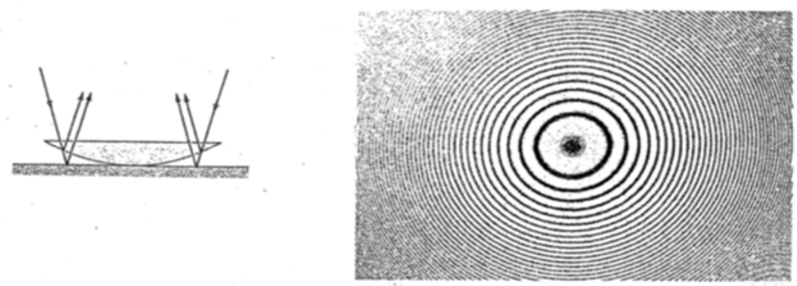

Fig 4

-Young used this experiment to ***prove*** the ***phase shift by π*** during reflection onto media with higher "density" in case of light waves. He assumed the ***presence*** of an ***infinitely thin (δ~0) film of air*** between *contact points* such that $\Delta\varphi_p = 0$ and the only $\Delta\varphi_r = \pi$ due to reflection ***from air on the lower boundary*** constitutes the total phase shift *between two waves that interfere in reflection*. This assumption explains the existence of a destructive interference in the reflected light for central position. To confirm this hypothesis he:

1. Used two different glasses for lens and plate such that $n_{lens} < n_{plate}$;
2. *Substituted* the air film by oil with refraction coefficient $n_{lens} < n_{oil} < n_{plate}$;

By this combination, he added a π-phase shift to the upper boundary reflection. As the total phase difference due to the reflections between the two rays becomes 0 (π-π) a bright central spot must appear. Young proved this experimentally. (Homework Ex.37.5)

MICHELSON INTERFEROMETER

-Michelson interferometer is an optical device used for very precise optical measurements. Its function is based on the principle of interference from thin films. The figure 5.a presents the principal scheme of such a device.

-A ray of light from a monochromatic source strikes the beam splitter **C** (glass plate coated by a very thin silver surface on right side). Part of the light (**ray 1**) passes through the silvered surface of **C**, through the compensator plate **D** and is reflected from mirror $\mathbf{M_1}$. In return, it passes anew through **d** and then is reflected from silvered surface of **c** to the observer. The other part of indecent light (**ray2**) is reflected from silvered surface at point **P**, and then reflected from mirror $\mathbf{M_2}$ and gets to the observer after passing through **C** plate. The compensator **D** has the same thickness as **C** and ensures that rays 1 and 2 pass the same path lengths inside glass. $\mathbf{M_2}$ is a movable mirror.

-Before starting measurements, one puts $L_1 = L_2$ and verifies that mirrors $\mathbf{M_1}$ and $\mathbf{M_2}$ are normal to rays 1 and 2. In this situation, the virtual image of mirror $\mathbf{M_1}$ (formed by reflection on silver) superposes to mirror $\mathbf{M_2}$.

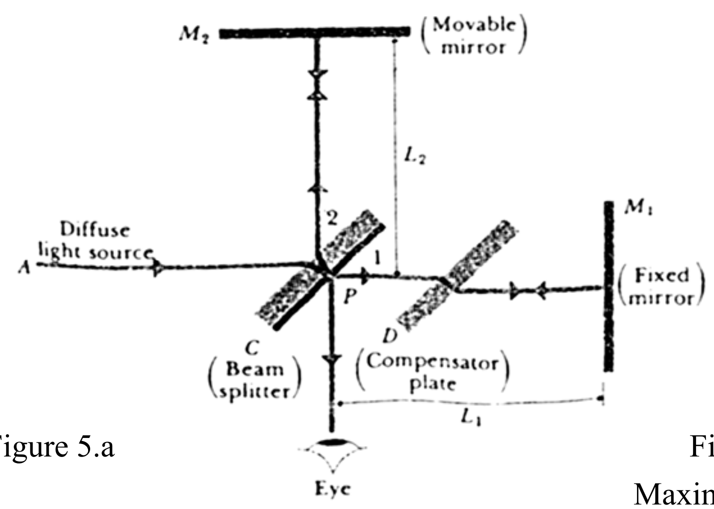

Figure 5.a

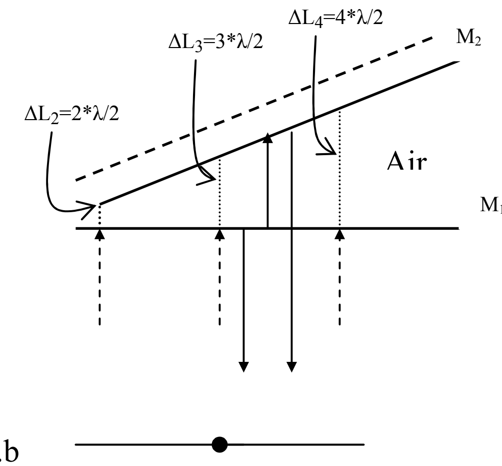

Figure 5.b

Maxima of order $M = 2$ $\quad M = 3 \quad M = 4$

- Then, one builds a "*wedge-shaped thin film of air*" for the couple of rays that reflect on mirrors $\mathbf{M_2}$ and $\mathbf{M_1}$ by a small rotation of $\mathbf{M_2}$ (fig 5.b) . The reflected light waves from these two boundaries interfere at the observer's eye and form on his retina *straight fringes of equal thickness*.

Note that:

1. The relative phase shift due to reflected waves is zero (**2π-2π**); So, $\Delta\Phi_{tot} = \frac{2\pi}{\lambda}\delta_{1-2}$

   and, as $\delta_{1-2} = 2(L_2 - L_1)$ it comes out that $\Delta\Phi_{tot} = \frac{2\pi}{\lambda} * 2(L_2 - L_1)$

   The maxima condition; $\Delta\Phi_{tot} = M * 2\pi; \Rightarrow\Rightarrow \frac{2\pi}{\lambda}(2\Delta L) = M 2\pi$ gives $\Delta L = M\lambda / 2$ (17)

2. One uses an extended monochromatic source to cover a good portion of mirror surfaces.
3. One uses a telescope to observe several (5-6) fringes simultaneously in view field.

- The observer starts measurements by fixing a fringe (*say bright of order- M*) on the crosshair of the telescope. Then, he moves "up" the mirror $\mathbf{M_2}$. When $\mathbf{M_2}$ is shifted by "$y = \lambda/2$", L_2 and ΔL are increased by $\lambda/2$ simultaneously for "***any air wedge thickness***". The next order ($M+1$) bright fringe substitutes the central bright fringe (M-order). If the mirror M_2 is shifted up by only "$y = \lambda/4$" , the next dark fringe will is substitute the bright fringe centered on the crosshair. ***So, the minimum length (equal to shift y) measured clearly by this device is λ/4 (Ex. if λ = 500nm*** y_{min}***= 500/4=125nm).***

During the measurement procedure, one counts the number '***nb***' of dark and bright fringes that pass through the crosshair of microscope while the mirror M_2 moves "up". As for ***nb = 1*** the displacement of mirror M_2 is $y = \lambda/4$ for a number "***nb***" of passing fringes(*dark&bright*) the ***displacement*** of M_2 is

$$y = nb * \frac{\lambda}{4} \qquad (18)$$

- There are ***two basic situations*** of measurements:

a) One knows the precise value of λ and calculates the displacement "y" of M_2 by use of (18). In these situations one is interested for ***precise measurements of lengths*** (ex. standard meter).

b) One uses the Michelson interferometer for ***precise measurements of λ***. ($\lambda = 4 * y / nb$)

A particular use of Michelson interferometer concerns the measurements of refraction coefficient "n" in gazes and it is explained in example 37.5 (H. Benson textbook).