WAVE OPTICS

GENERAL

-The "ray optics" cannot explain the results of the two following experiments;

a) When passing through small openings or hitting small obstacles, the light "**bends around borders**" and reaches inaccessible regions for optical rays….DIFFRACTION
b) The superposition of two or more light beams in "*some special conditions*" produces fringes of maxima and minima intensity on a screen….INTERFERENCE

Diffraction and interference are ***typical wave phenomena*** and we will introduce them through TW their action in mechanical waves (TW and LW- sound waves). Then, we will apply the same model and its results to understand the ***wave behaviour of light***.

- Research showed that the *visible* **light** is *just one* ***part of electro-magnetic waves*** (radio&TV waves, x & γ rays are part of ***E.M***. spectrum, too).

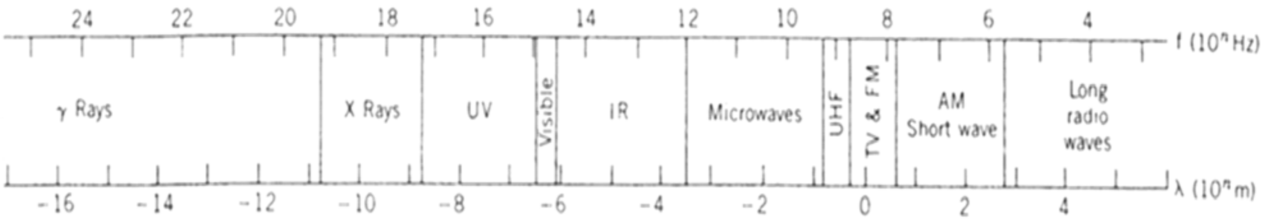


Fig.1a The electromagnetic spectrum

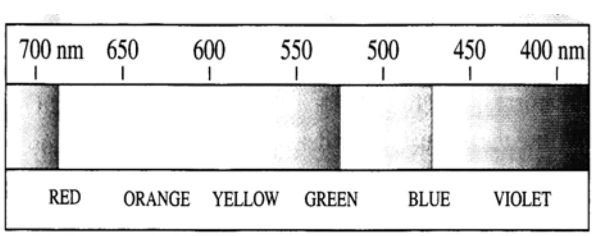


Fig.1.b Visible Light

-The velocity of ***E.M.*** waves (light included) in vacuum is **c = 3*10^{8}m/s**. When light propagates in a transparent material medium, this velocity decreases to $v = c/n$ **(1)** where n is the **refractive index** of the medium ($n > 1$). So, if the wavelength of light in vacuum is $\lambda_0 = T * c$, when propagating in a medium its wavelength becomes $\lambda_n = T * v$. The period ***T*** and the frequency ($f = 1/T$) of light wave remain constant but the wavelength decreases $\lambda_n / \lambda_0 = v / c = \frac{1}{n} < 1$

Remember that ; $\lambda_n < \lambda_0$ **and** $\lambda_n = \lambda_0 / n$ ***(2)***

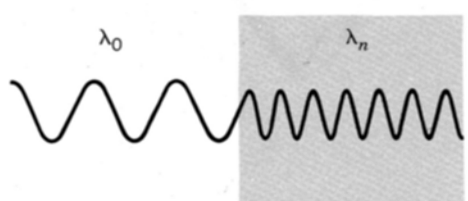


Fig.1.c The *frequency remains the same but the wavelength decreases inside matter*

DIFFRACTION

- The ***optical rays*** show ***the path*** followed by "*light particles*" that are figured out as "*energy droplets*" that transport light energy. So, the ***rays show the direction of energy flow***. In a ***wave model*** the *energy propagates* ***perpendicular to wave fronts***. So, it comes out that ***the rays*** *must always be* ***perpendicular*** *to the* ***wave fronts***.
- For a point source, the wave fronts are spherical and rays follow the radius directions. At a big distance from the point source, a portion of wave fronts can be fitted by a plane. Being perpendicular to wave front, the rays are perpendicular to these plane wave fronts.

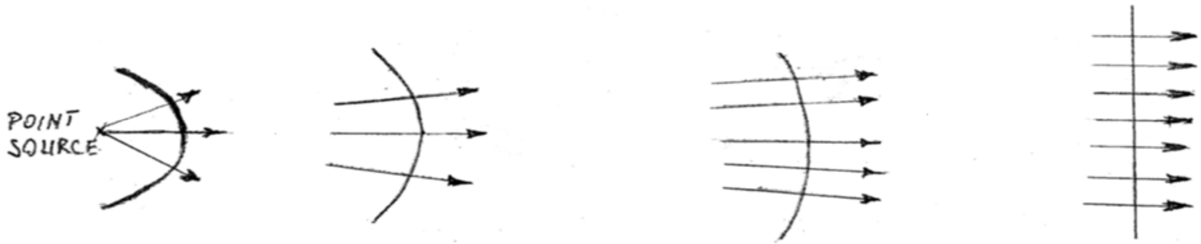


Figure 2. The ***beam of*** **rays** describes the ***direction of energy flow*** and it is ***normal*** to the ***wave fronts***.

-When a **plane wave** hits a large opening (or obstacle), there *is no **wave motion** out the space region covered by rays* (figure 3.a).But, when the opening (or obstacle) ***dimension*** decreases beyond a certain limit***(~10λ)***, the wave motion gets inside zones that cannot be reached by rays. This phenomenon is known as ***diffraction*** and can be *explained only by **wave fronts concepts** and **Huygens' Principle***.

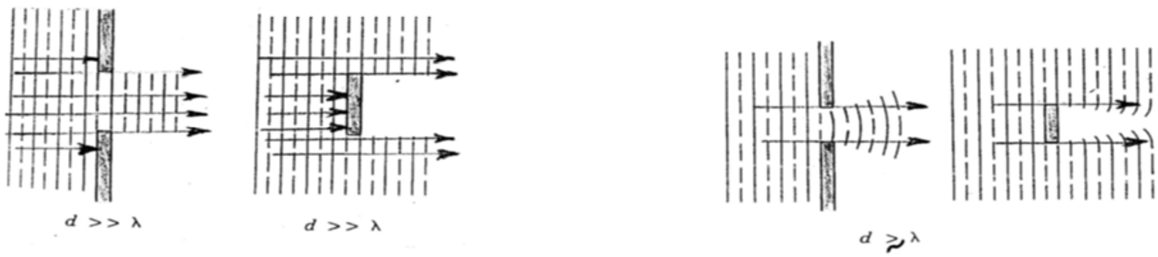
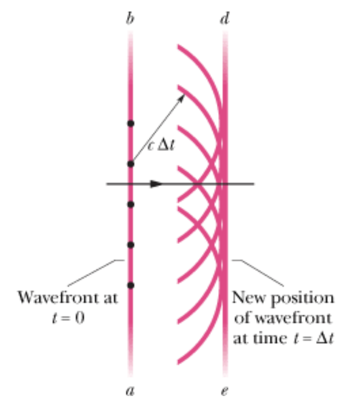


Figure 3. a) - "Ray_normal" situation (d >>λ) b)- "Ray_ abnormal" situation (d >λ)



Huygens principle: ***Each point of the wave front is a source of secondary waves. The new wave front is the envelope of these secondary waves***.

This principle explains all situations; large or small openings and obstacles;

a) ***Large*** openings (or obstacles). There are a big number of points that contribute to build the new wave front. The common envelope profile makes invisible the border effect of secondary waves (wavelets) and the result is a new ***almost plane wave front*** (fig. 4a)

b) ***Small*** openings (or obstacles). Only a limited number of points contribute with their wavelets. The common envelope cannot cancel the curved parts of due to the secondary waves and the result is a new ***curved wave front(*** fig.4.b).

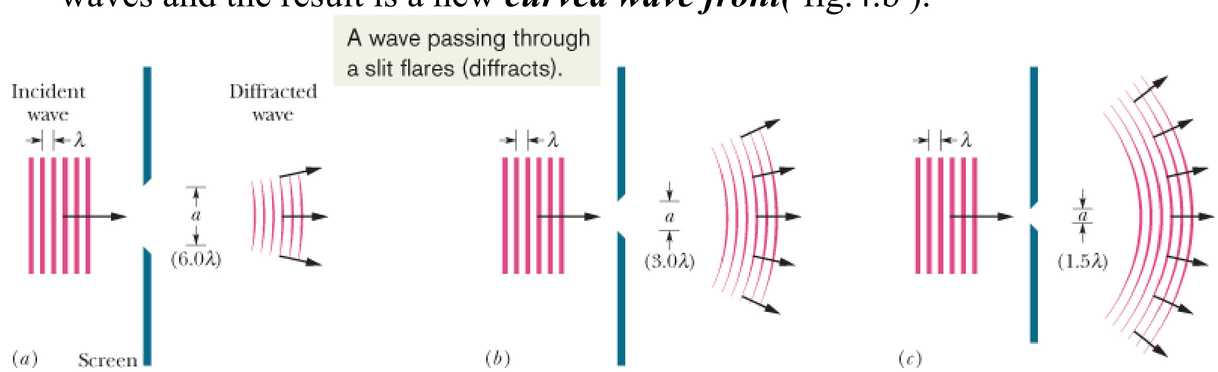


Figure 4. a- large number of wavelets (d >λ) b- only one wavelet (d ≈ λ)

- When dealing with *essentially wave phenomena*, like sound, the ***concept of wave front*** is sufficient to ***explain all propagation situations***. ***The ray concept*** *does not offer any important advantage in the case of* ***pure wave*** *phenomena and it is* ***used very rarely***.
Note: Diffraction is a common situation for sound waves because $\lambda \sim$ ***"openings order"***.
Ex.; for $f = 150Hz$ (**low sound frequencies**) the wavelength is $\lambda = 340m\ s^{-1}/150\ s^{-1} = 2.2m$.

- In the case of light, the ***diffraction*** effects appear when it propagates through ***very small openings*** because λ -values are ***very small***. But, the mechanism of diffraction is the same because *Huygens principle applies the same way no matter the physical nature of wave*.

INTERFERENCE

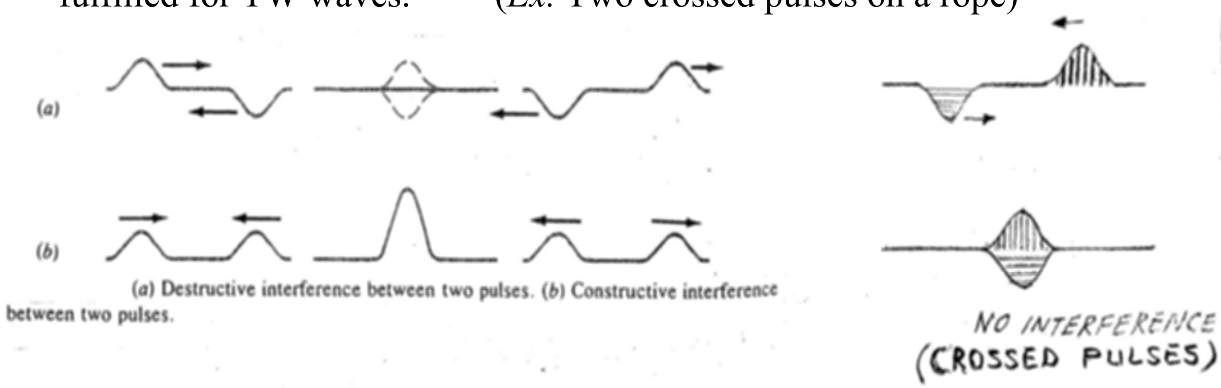
- *The principle of linear superposition:* If the paths of two (or more) waves propagating in a medium cross to each other, then the total "***displacement***" at crossing point is equal to ***the sum of displacements*** due to each wave ***at this point*** of medium.

$$y_T = \sum_{i=1}^{n} y_i ; \quad i = 1,2,... \qquad (3)$$

This principle is ***valid no*** *matter what is* ***the physical nature "displacement"*** .

- When two waves pass by the same point in medium, there is ***always a superposition*** but ***not always interference***. There is interference when the waves:

a)Have "***displacements***" along the ***same space direction***. This condition is not always fulfilled for TW waves. (*Ex.* Two crossed pulses on a rope)

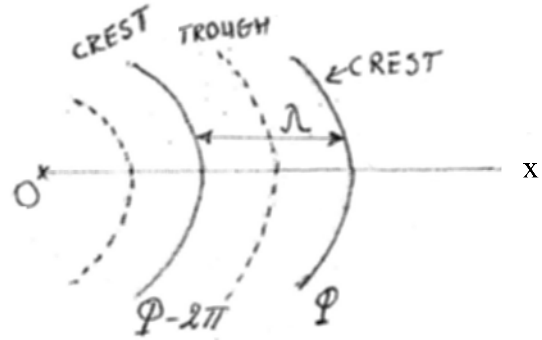


(a) Destructive interference between two pulses. (b) Constructive interference between two pulses.

Figure 5

b)Have the ***same frequency***. This requirement is ***essential*** so that one may ***observe a stable interference.*** To observe interference, one must deal with a ***standing wave***.

-The ***wave front*** is the locus of the ***medium points*** at which the wave function has the ***same phase value*** Φ. The consecutive wave front oscillates " *in phase*" with the first one. So, it is the locus of medium points at which the phase is Φ +/- 2π.



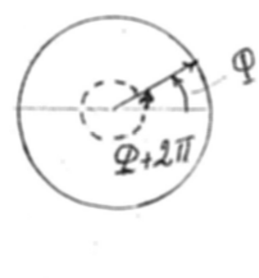


Figure 6.a

Figure 6.b ***Phase shift 2π*** between any two points of medium on the directions perpendicular to wave front at a ***distance λ***

The distance between these two consecutive wave fronts is ***2π.*** We chose a direction Ox as shown in figure 6.a and find out that:

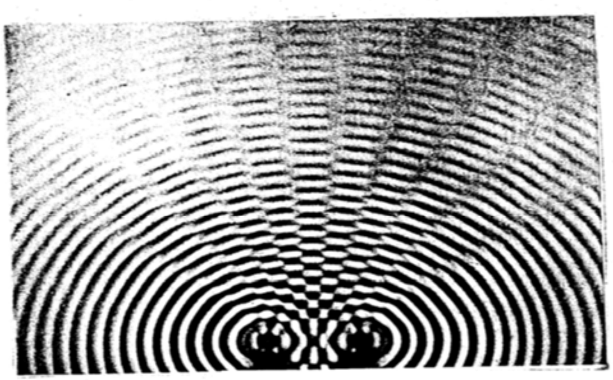
$$\Phi_1 = kx_1 - \omega t; \_ \Phi_2 = kx_2 - \omega t;$$

$$\Phi_1 - \Phi_2 = k(x_1 - x_2) = \frac{2\pi}{\lambda}\Delta = 2\pi \rightarrow \Delta = \lambda \qquad (4)$$

So, ***any two consecutive wave fronts are at a distance λ to each other***.

<u>INTERFERENCE ON A TWO DIMENSIONNAL SPACE (plane surface</u>)

-The standing waves along a string are an example of ***1D space*** interference. A ***2D space*** interference can be built by using two plungers that vibrate with the ***same frequency*** and ***phase*** on the still surface of a water tank (fig 7).



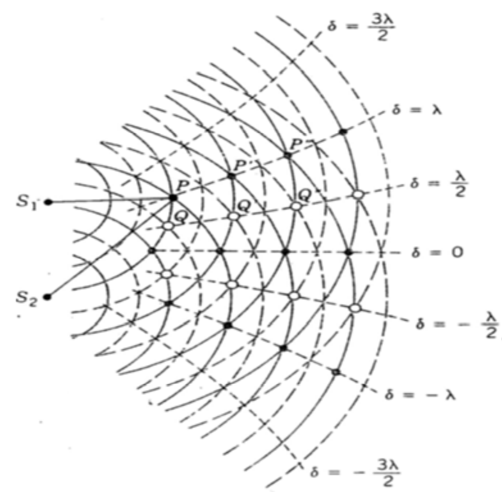


Figure 7

Figure 8

- The ***wave model*** explains perfectly the produced picture (fig.7) by using the ***superposition principle,*** two point sources (S_1, S_2) that oscillate with ***same frequency, same amplitude*** *and* the *same space direction* of "***displacements***" for the two waves that superpose.
Actually, one can easily figure out that;

There is CONSTRUCTIVE (***double amplitude***) interference at any point where:

- a **crest** *wave front* meets another **crest** *wave front*.
- a **trough** *wave front* meets another **trough** *wave front*.

There is DESTRUCTIVE interference (***zero amplitude***) whenever

- a crest wave front meets trough wave fronts or vice versa.

-Although the two waves propagate continuously on water surface, there are directions (***lines***) of ***zero displacement*** and there are directions (***lines***) of ***maximum displacement***. So, *one can derive that* ***the interference status depends on the location***.

-To understand this, one must refer to that fact that the status of interference at a given point depends on the ***phase shift between the two waves that superpose at this location***.
If this point is at distance r_1 from source S_1 and r_2 from source S_2, the phases of the first and second wave at its location are $\Phi_1 = kr_1 - \omega t \_and\_ \Phi_2 = kr_2 - \omega t$. Then, the phase difference between the two waves that superpose is $\Delta\Phi = \Phi_2 - \Phi_1 = k(r_2 - r_1) = k\delta$ **(5)**

As $k = 2\pi/\lambda$ is a constant, the **interference status** depends only on the **path length difference** δ between the two waves at this point. $\delta = r_2 - r_1$ (6)

r_1 is the distance from the first source; r_2 is the distance from the second source;

- One may easily observe that the ***phase difference*** :

1a) is zero($\Delta\Phi = 0$) for each point located on central line because $r_1 = r_2$ and $\delta = 0$.
The two waves arrive with the same phase; **constructive interference**.

1b) is $\Delta\Phi = \pm 2\pi$ (**constructive interference**) **also** at all points where $\delta = r_2 - r_1 = \pm\lambda$ **.**
Those points are located on two hyperbola wings.

2) Between the *central line* and *each of these wings* of *constructive interference* there is a *line of* ***destructive interference*** at points where $\Delta\Phi = +/-\pi$ or $\delta = r_2 - r_1 = \pm\lambda/2$.
Those points are located on two hyperbola wings ($\delta = +/- \lambda/2$).

Remember: there is a ***maximum displacement*** on points of ***constructive interference*** and ***zero displacements*** at points of ***destructive interference***.

- **In general**, on each point where the two waves meet with a ***path length difference***:

$\delta = M\lambda \_ M = 0, \pm 1, \pm 2, ...$ there is **constructive** interference. (7.a)

$\delta = (2m+1)\lambda/2; \_\_ m = 0, \pm 1, \pm 2, ...$ there is **destructive** interference. (7.b)

-***IMPORTANT*** : The derivation of conditions(7.a- b) is based only on **phase difference** requirements. So, they are valid for the interference ***of any two waves no matter their physical nature, light waves included***. The same rules explain bad or good ***sound*** hearing at different positions inside the same concert hall and also the low or high signal reception of radio and TV signals when the antenna in located at different positions.

YOUNG'S EXPERIMENT

-To show the wave nature of light Young used the schema of figure 9. In this scheme:

- Two ***sources*** of "in phase " oscillations are two *secondary narrow slits* S_1, S_2 equidistant from the primary source (pinhole receiving sun light).
- The considerable distance from pinhole give ***plane waves at slits input***.
- Very narrow slits produce ***cylindrical wave fronts***.
- The model screen "far from" slits simplifies expression for ***path difference.***
- Using a coloured glass one get ***monochromatic light*** (***one*** λ) .

Nowadays, one gets easily the monochromatic plane wave by using a laser source.

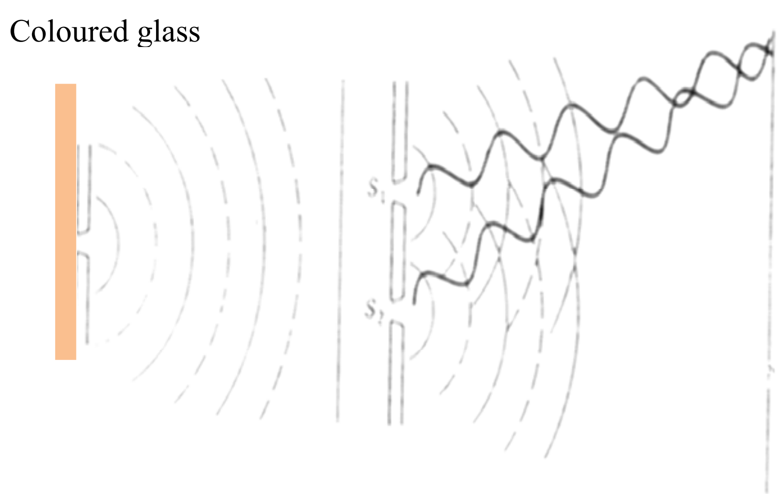


Figure 9

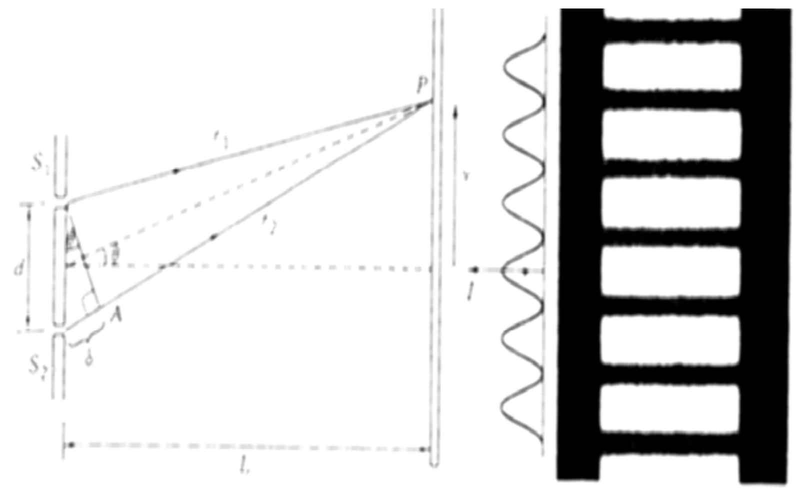
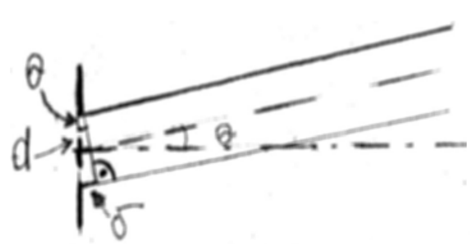


Figure 10

-The ***geometrical path length difference*** for a *given point* P on the screen (fig 10) is $\delta = r_2 - r_1$
As ***L >> d***, the two paths are *almost parallel* and from fig.11, we find that

$$\delta \cong d\sin\theta \qquad (8)$$



d- distance between two narrow slits
θ- the angle that defines the point location on screen

Figure 11

-Knowing that there is *constructive interference* for $\delta = \pm M\lambda$ and *destructive interference* for $\delta = \pm(2m+1)\lambda/2$, we find the corresponding directions (***angles)*** for fringes of ***maximum*** intensity by the expression (9) and ***minimum*** intensity by (10)

$$\sin\theta = M\frac{\lambda}{d} \quad M = 0, \pm1, \pm2.. \quad (9)$$

$$\sin\theta = (2m+1)\frac{\lambda}{2d} \quad m = 0, \pm1, \pm2.. \quad (10)$$

Notes: a) For M = 0 one gets the **central maximum (** in the middle of interference pattern)**.**
b)The first minimum "upside($\theta > 0$***)" the*** *central maximum corresponds* ***to m = 0.***

-These ***interference fringes*** are parallel to slits. The location of maxima fringes close to the central region where **θ is small** (***θ≅tgθ≅sinθ***) can be found (see fig 10) as follows:

$$\frac{y_m}{L} = tg\theta \cong \sin\theta \Rightarrow\Rightarrow\Rightarrow \frac{y_m}{L} = m\frac{\lambda}{d} \Rightarrow\Rightarrow y_m = m\frac{\lambda}{d}L \qquad (11)$$

- *Two sources that oscillate with the* ***same frequency (f)*** *are said to be* ***<u>coherent sources.</u>*** *By using a* ***phasor diagram****, one may see that* ***the phasors of <u>two coherent sources</u>*** *have a* ***<u>constant phase difference</u>*** *between them* ***;*** $\Delta\Phi_{1,2} = [(\omega_2 t+\varphi_2) - (\omega_1 t+\varphi_1)] = \varphi_2 - \varphi_1$.
Two coherent sources (***same f***) oscillate ***<u>in phase</u>*** if $\Delta\Phi_{1,2} = M*2\pi$ where $M=0,\pm1,\pm2,..$
In the upper scheme S_1, S_2 are two ***coherent sources*** and ***in phase*** because $\Delta\Phi_{1,2} = 0$.
If $\Delta\Phi_{1,2} \neq M*2\pi$ the two sources are ***<u>out of phase</u>*** . Often this nomination if used for two sources in ***opposite phase*** ; $\Delta\Phi_{1,2} = (2m+1)\pi$ where m = 0,± 1 ,±2,..

- ***Remember***: The ***interference status*** is decided by the phase shift $\Delta\Phi = k\delta$. If the two waves propagate in air, the refraction coefficient is " ***n =1" and*** $k=k_0 = 2\pi/\lambda_0$. When the two waves propagate in another medium *"* ***n >1"*** *and* $k_n = 2\pi/\lambda_n = 2\pi/(\lambda_0/n) = n*(2\pi/\lambda_0) = n*k_0$

So, $$\Delta\Phi = k_n\delta = nk_0\delta = k_0 n\delta = k_0\delta_n \qquad (12)$$

The ***<u>optical path length difference</u>*** when the waves propagate in the same medium quantity is $\delta_n = n*\delta$ (13)

If *they propagate in two different mediums,* $\delta_n = n_2*r_2 - n_1*r_1$ (14)

In all situations***, the <u>phase shift due to path length difference</u> is*** $\Delta\Phi_{path} = k_0\delta_n$ (15)

<u>THE INTENSITY OF INTERFERENCE PATTERNS</u>

-The use of very narrow pinhole and the big distance to slit justifies the plane modeling for wave fronts at the slits input which means ***equal amplitudes and equal phase at for the two waves at the sources(slits)***. By neglecting the slight differences related to unequal path lengths, these two waves have equal amplitudes at each point on the screen.

- Then, at *<u>any given point</u> on the screen*, the " light ***displacement***"
  produced by the source S_1 is $E_1(t) = E_0 \sin\omega t$ (16)
  produced by the source S_2 is $E_2(t) = E_0 \sin(\omega t + \varphi)$ (17)

E_0 - same amplitude as equal sources produce equal amplitudes everywhere on screen
ω - same frequency (first requirement for coherence)
φ - ($\Delta\varphi_{1,2} = \varphi$) ***constant in time*** *which is the* requirement for stable coherence.

-The *phase shift* **φ** in (16) is defined by the point ***position*** and does not change in time because the waves are coherent. The *phase shift* is fixed by the *path length difference* **δ** *at that point as* $\varphi = (2\pi/\lambda)*\delta$

-The principle of linear superposition gives the total " *displacement*"

$$E_T = E_1 + E_2 = E_2 + E_1 = E_0 \sin(\omega t + \varphi) + E_0 \sin(\omega t) = 2E_0 \cos\frac{\varphi}{2}\sin(\omega t + \frac{\varphi}{2}) \quad (18)$$

The amplitude of total "displacement" is $A_T = 2E_0 \cos\frac{\varphi}{2}$ (19)

-At this point we remember that the total energy of an SHO is $E = U_{max} = \frac{1}{2}kA^2 \approx A^2$

So, the ***power*** (*energy/time*) of light oscillations at the point on the screen is proportional to A^2 and the **intensity** (***Power/Area***) is proportional to A^2. This means that the ***evolution of light intensity*** on different points on the screen will be the **same as the evolution of** A^2. To simplify the follow up of intensity on the screen we note $I = A^2$. If only one source sends the light at a point P on the screen, the *light intensity* at P-point is $A^2 = E_0^2$, so we have $I_0 = E_0^2$. In presence of the light from the second source, the intensity at P-point becomes

$$I = A_T^2 = 4E_0^2 \cos^2\frac{\varphi}{2} = 4I_0 \cos^2\frac{\varphi}{2} \quad (20)$$

The graph of this function for different locations of considered P-point (*i.e. different δ-values and different phase difference*) is presented in figure 12. Its maxima correspond to the positions where $\delta = \pm M\lambda$ or $\sin\theta = \pm M\lambda / d$.

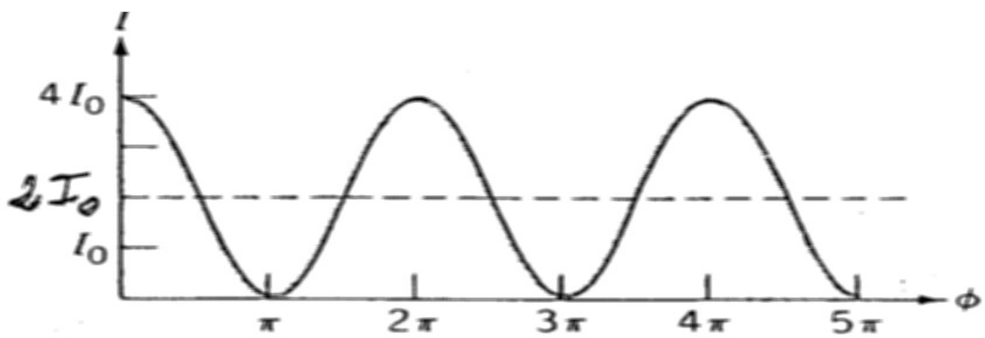
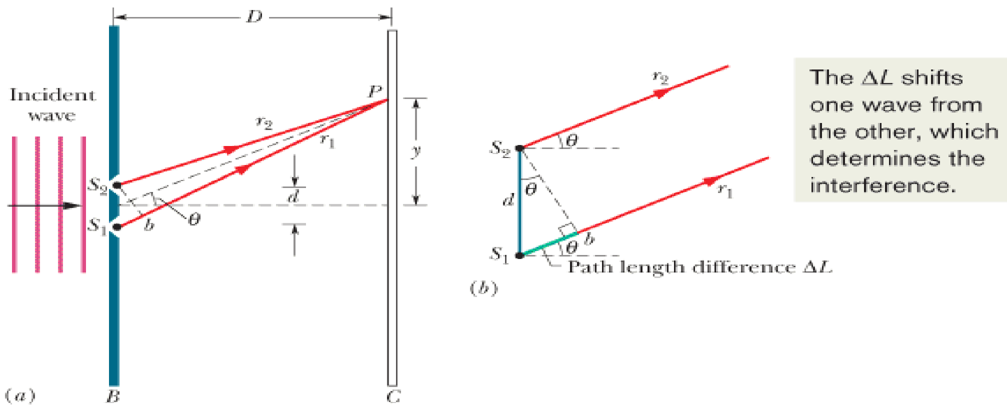


Figure 12

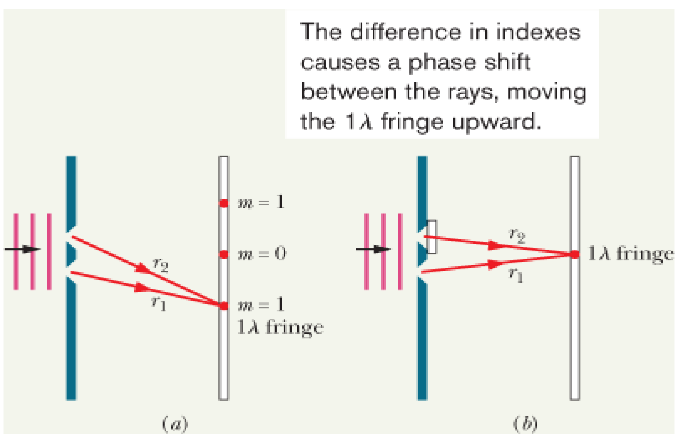
-Note that, if the waves would not interfere, they would simply superpose and the light intensity at considered point would be $\boldsymbol{2I_0 = 2\,E_0^2}$. Due to the presence of interference, at the same point, the ***light intensity***:

a) ***increases twice*** ($\boldsymbol{4*I_0 = 2*(2\,I_0)}$) if the considered point is at a ***maximum location***
b) is reduced to ***zero*** if the considered point is at a ***minimum location***.

Important Note: The ***interference produces only energy redistribution*** but the average (**2*I**) ***intensity*** and ***the total energy on the screen remain unchanged***.



Exercise: λ_0 = 600nm. Plastic transparent strip with n =1.5 over one of slits.
a) Locate it on one of the slits so that the pattern shifts upward.
b) Find its thickness "L" so that the pattern moves up by 1 inter fringe distance.



a) Let's refer to the central maximum where the interference order is M = 0 without the strip. At this point, the phase shift is initially $\Delta\varphi = k\delta_{op} = (2\pi/\lambda)*(r_2-r_1) = M*2\pi = 0*2\pi = 0$
In presence of the strip at upper slit, the optical path length of the second wave increases. This brings to δ_{op} *increase* and consequently to *increase* of phase shift Δφ. It comes out that the maximum of order M=+1(*down*) takes the place of central maximum M= 0 which shifts up.

b) Let's refer anew to the central maximum, where the interference order is M = 0 *without the strip*. At this location, the phase shift between the two waves is initially
$\Delta\varphi = (2\pi/\lambda)*(r_2-r_1) = M*2\pi = 0*2\pi = 0$

At the <u>*same place*</u>, <u>*in presence of the strip*</u>, the phase shift between waves becomes
$\Delta\varphi_1 = (2\pi/\lambda)*[(r_2 - L + nL) - r_1] = (2\pi/\lambda)*[(r_2 - r_1) + (nL - L)] =$
$= (2\pi/\lambda)* (r_2 - r_1) + (2\pi/\lambda)* L(n - 1) = \Delta\varphi + 2\pi = 0 + 2\pi$

So, $(2\pi/\lambda)* L(n - 1) = 2\pi$ and $L = \lambda/n-1 = 600/(1.5-1)= 600 / 0.5 =1200nm$
Finally we get $L=1.2*10^{-6}m = 1.2\mu m$