16.MECHANICAL WAVES

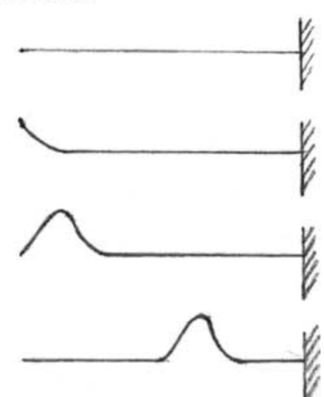
GENERAL

- SHO is a periodic motion in time (i.e. constant T-period, f-frequency and ω-circular frequency) and a constant amplitude A(length, voltage, density,..).
  SHM is one SHO where A[meters] is always a real displacement.
- An SHO or SHM is a particular type of wave; it is a wave which is continuing without end in time, . In general a wave is the propagation of a disturbance through a medium. The disturbance is built up in a restricted area and then it is propagated through all parts of a medium(Ex. Ripples produced in a point of a lake, sound in air, radio and TV waves ..).
- Note that during wave propagation each particle of medium returns to its initial position after transporting the "disturbance" to the next particle. In a wave motion there is no mass transport.
- Two types of waves:
  a) Mechanical (material medium that transports waves have restoring proprieties.
  b) Electromagnetic (waves are propagated in matter or vacuum medium).

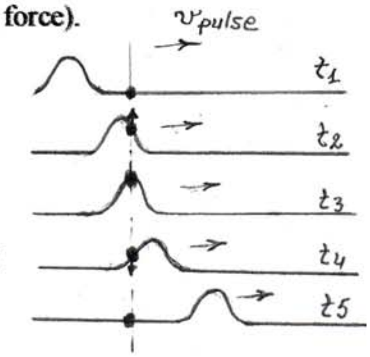
In this chapter we shall see the mechanic waves

16.1 WAVE CHARACTERISTICS

- Consider a string with a fixed end. A short disturbance produced at the free end propagates along the string. Each point of the string repeats the displacement of the free point and then returns to the equilibrium. **This is a wave pulse**.
- Note that each particle of the string moves along a direction that is perpendicular to that of wave propagation. **This is a transverse** wave. When each particle of medium moves along the propagation direction of wave we have a **longitudinal wave.** Example: Break liquid particles when we give a short break. There is a short increase in liquid density that is transported to the breaks. When we release the liquid density returns to normal value).



- A solid medium can propagate transverse and longitudinal waves due to its three dimensional structure(restoring forces along 3 dimensions). A gas medium can propagate only longitudinal waves. A liquid transport longitudinal waves in volume and transverse waves in surface (the surface tension plays the role of a restoring force).



- The motion of medium particles is a small displacement around the equilibrium. There is no matter transport along wave propagation direction.

- The disturbance profile defines the wave function. The amplitude of this wave function is displacement for the string wave, pressure for the sound or liquid waves. Note that it may be a vector (displacement) or scalar(pressure) and the wave function is not any more a simple sin function as in the case of SHM.

- What is transported during mechanical wave propagation? We can consider a leaf on a quite water surface. It has a potential energy E_o. During a pulse wave propagation, the leaf moves and gains a kinetic energy E_k. When the pulse is far the leaf returns to its initial position and has the same energy E_o. So, no energy remains to the leaf; there is just a energy transport. The same assertion is valid for the water particle under leaf. The wave propagation is associated with kinetic energy transport. As the momentum is related to kinetic energy as $E_k = \frac{mv^2}{2} = \frac{m^2v^2}{2m} = \frac{p^2}{2m} \rightarrow \left|\vec{p}\right| = \left|\sqrt{2mE_k}\right|$ we derive that during wave propagation there is a momentum transport, too. **<u>One wave motion transports ENERGY and MOMENTUM from one point to the other point of a propagating medium.</u>**

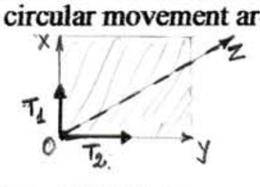
16.2 WAVE SUPERPOSITION

- Suppose that in each of two ends of a long tensioned string is built up simultaneously a pulse wave. The two pulses travel along the string and at given moment they overlap. There is a wave **SUPERPOSITION** along string.
- What happens with medium particles during wave superposition? Here it works the PRINCIPLE OF LINEAR SUPEPOSITION:
If the waves $y_1, y_2, y_3, \ldots y_n$ propagate simultaneously in the same medium the total displacement of each medium particle is

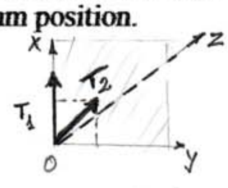
$$y = y_1 + y_2 + y_3 + \ldots\ldots + y_n \quad (1)$$

Notes: 1) If y_i are mechanical displacements, eq. (1) is a vector sum;
2) If y_i are scalars(ex. air density) it is a algebraic sum.
3) It is valid only in the limits of Hook's law validity (small y_i).

- When two or several <u>scalar waves</u> superpose it is produced <u>always</u> a special phenomena called INTERFERENCE. In this case the medium particles displacements increase "<u>CONSTRUCTIVE INTERFERENCE</u>" or decrease (or remain at equilibrium) <u>significantly</u> their displacements and there is "<u>DESTRUCTIVE INTERFERENCE</u>".
- When dealing with superposition of <u>vector waves</u> the interference is possible only if the displacement vectors are parallels or have components following the same space direction. If displacement vectors are perpendiculars there is wave superposition but not interference. In this case medium particles perform an elliptical or circular movement around their equilibrium position.



NO INTEFERENCE

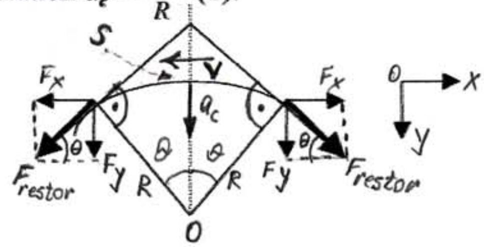


INTERFERENCE

16.3 PULSE ON A STRING; SPEED CALCULATION

- Small pulse propagation along a tensioned string. Consider so small that the restoring force magnitude $|F_{restor}|$ (string tension) is constant.
- In the laboratory frame the pulse moves right but do not forget that spring particles do not move right or left at all(board drawings).
- To simplify the study we use a pulse related frame. In this frame the pulse (and string profile) are fixed but the string particles move left with constant speed $-V_{pulse}$. Note that we are yet in an inertial system and do not need to make correction to the II law of Newton.
- As string particles "are moving along a curved profile" with tangential speed v, there is a centripetal acceleration $a_c = \frac{v^2}{R}$ (2).

The restoring force is the sum of two string tensions following Oy axis. So, from the figure we have



$F_y = F_{restor} \sin\theta; F_{tot} = 2 * F_{restor} \sin\theta$ (3)

For small angles $\sin\theta \approx \theta$ and

$F_{tot} = 2 * F_{restor} * \theta$ (3')

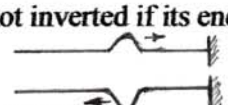
- Now we apply the Newton's law for he resultant force acting on string and have: $F_{tot} = ma; \Rightarrow 2*F_{restor}*\theta = \frac{mv^2}{R}; \Rightarrow v = \sqrt{\frac{2*\theta*F_{restor}*R}{m}}$ (4)
- We express the mass of spring piece as function of its length s(m) and its linear density μ (Kg/m) $m = \mu * s = \mu \cdot 2R\theta$ and we get

$$v = \sqrt{\frac{2R\theta*F_{restor}}{2R*\theta*\mu}} = \sqrt{\frac{F_{restoring}}{\mu}} \qquad (5)$$

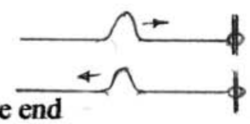
This result is general: **the propagation speed of mechanical waves in a medium is ~ to SQRT of restoring force and inverse ~ to SQRT inertia.**

16.4 PULSE REFLECTION AND TRANSMISSION

- A pulse traveling along a string during a reflection
  a) is inverted if its end is fixed (III Newton)
  b) is not inverted if its end is free.

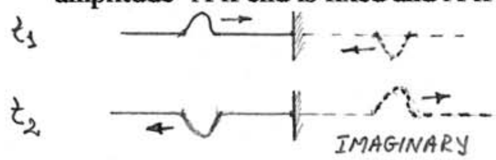


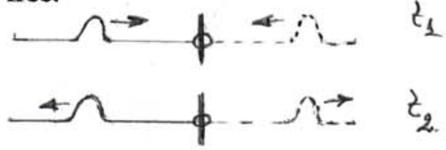
Fixed end



Free end

- To built the string profile for each moment we can use un imaginary wave that propagates with same sped in inverted direction with amplitude -A if end is fixed and A if end is free.





- In two above presented situations there is only one secondary wave; the reflected wave. If another string is fixed to the end of the first one, two secondary waves appear; a reflected one and a transmitted one. As the tension (F_{restor}) is the same along the two strings, their linear densities define the relative velocity $v_2 / v1 = \sqrt{\mu_1 / \mu_2}$ (E_{tr}/E_{ref}, L_{tr}/L_{ref})