REMEMBER

-The ***apparent colour*** of thermal radiation depends ***only on the temperature of "source" object.*** The materials with ***different constituency*** show the ***same apparent colour*** at the ***same*** (high) ***temperature.***

- The **black body** is an ideal system that absorbs all radiations incident on it and serves for studying the thermal radiation**. The** energy density **u(T)**[J/m^3] of radiation is the quantity of radiation energy contained into 1 m^3 of space. The distribution of **u(T)** between different wavelengths is given by **u(T)$_\lambda$** *spectral energy density* **u(T)$_\lambda$** [J/m^4]. One can measure **u(T)$_\lambda$** through spectral analysis.

-The **Wien's first law** gives the relation between the *temperature* and *maximum position;* $\lambda_{\max} * T = 2.898 * 10^{-3} [mK]$

- Planck hypothesis about "**ε**" $\varepsilon = h * f;\_where\_ f = c/\lambda \quad h = 6.626 * 10^{-34} J * s$

produced the result $u_\lambda(T) = \dfrac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$

-The photoelectric effect is the emission of electrons by an object when a beam of light illuminates it. The maximum velocity of ejected electrons is

$$\frac{1}{2} m v_{\max}^2 = eV_0$$

-Einstein made a decisive hypothesis: ***Radiation behaves as if it were composed of collection of discrete energy quanta of magnitude each*** $E = h * f$

$$e * V_0 = h * (f - f_0)$$

- Compton effect is the scattering of ***X-ray photons*** by ***free electrons***. Photons with a different λ appear in the scattered radiation and λ value depends on the scattering **angle. Compton effect proves that x-rays behave as photons and have linear momentum**

$\lambda' - \lambda = \left(\dfrac{h}{m_0 c}\right)(1 - \cos\theta)$ ($h/m_{0*}c$) = 0.00243 nm (Compton wavelength.)

- The light is emitted during the transition of atoms from one level of higher energy into a level of lower energy. It is absorbed by an inverse transition. The wavelength of light related to a transition $1 \Leftrightarrow 2$ can be calculated by the relations

$$\varepsilon = E_1 - E_2 = \Delta E = hf;\_and\_\lambda = c/f$$

THE DUALITY WAVE-PARTICLE OF LIGHT

- A short review about the human concept about the nature of light shows that:

a) From the **antiquity till Newton** the **light** was seen as **stream of tiny particles**.

b) From the half of 19^{th} century till the beginning of 20^{th} century, the **light** was seen as **wave phenomena**. The experiments (Young , Fresnel…) and the theory (Maxwell, Huygens..) provided a full set of proofs for the wave nature of light.

c) During the first 30 years of 20^{th} century, appeared a number of experiments (thermal radiation, photoelectric effect, Compton effect, line spectra from atoms), which had explanation only in the frame of "light constituted by particles – photons".

So, it seems natural to ask: "What is the **true** nature of light; ***wave or particle***?"

- Note that the answer was not easy because physics is an experimental science and **there was experimental proof for the two points of view**.
At the beginning, the physicists followed this logical reasoning:

At ***low frequency*** electromagnetic wave region, a ***single photon*** posses very ***small amount of energy*** (***E = h*f***). In this range of energy (***radio & TV waves***) experiments deal with a **big number** (billions) of **photons**. ***Comment***; *We are not able to observe the behaviour of a single photon and what we see as a* ***wave*** *is the* ***collective behaviour*** *of a big number of photons*.

a) At ***high frequency*** electromagnetic wave region, a ***single photon*** posses ***high energy*** (***E = h*f***). In this range of energy (***X-rays***) all ***experiments deal with single events*** and *we are more precise* when we judge about the *nature* of electromagnetic *events*. Here we find out a **particle (*photon*) behaviour; so** this is the right nature of light.

- Based on the last comments, many physicists thought to ***get the last proof*** for particle thesis in the region of ***visible light***. The idea was the following; in this region of wave-lengths, we apply successfully the geometrical optics(***particle*** behaviour) and the wave optics(***wave*** behaviour). ***Assuming that the wave behaviour concerns only a big number of photons, the interference and diffraction patterns must disappear if we send the photons one by one at the input of Young's slits***. Many physicists performed the experiment of Young with very low light intensity but the interference patterns ***did not disappear***.

- These experiments brought to the result that ***the wave nature of light is not a feature that concerns only a big number of photons***. As result, the ***duality particle – wave comes out as an intrinsic feature of light. Even a single photon has wave characteristics***.
This type of behaviour appears clearly when we consider the photon in special theory of relativity. The total energy of a relativistic particle is

$$E^2 = p^2c^2 + (m_0c^2)^2 \qquad (1)$$

Applying equation (1) for one photon ($m_0^{ph} = 0; \_ E_{ph} = h * f; \rightarrow f = c/\lambda$) we get

$$E_{ph}^2 = p_{ph}^2 c^2; \rightarrow p_{ph} = E_{ph}/c; \rightarrow p_{ph} = h*(f/c) = p_{ph} = h/\lambda$$

$$p_{ph} = h/\lambda \qquad (2)$$

The relation $p_{ph} = h/\lambda$ correlates clearly a *particle* characteristic (***momentum)*** to a *wave* characteristic (*wavelength*) and they define the **same physical event** (light).

- It's true that some natural phenomena are explained **in full inside** particle model and some others are explained **in full inside** the wave model but the ***light can not be explained in full inside only one of those models***. In some experiments its behaviour is explained by the wave model and in some other by the particle model. Even ***a set of light characteristics cannot be supported by any of these models***.

Example: - It behaves as a wave "**with no need for a propagating medium**"!!

- It behaves as a particle which "**speed do not depends on the source speed**"!

Conclusion; "There is ***no sense*** to talk about the ***true nature*** of light". So physics uses:

a) the **wave model** for the **light propagation** and

b) Particle model – photons for the **absorption, emission and scattering of light.**

THE MATTER WAVES

-Bohr's theory used the "**allowed circular orbits**" (fig 1) to explain the origin of line spectra of hydrogen and other one-electron systems. Bohr model has two weak points:

a) It did not give any justification about the reason why other orbits were not allowed.

b) It was not able to offer any way for *calculating the intensities of spectrum lines*.

In fact, **the greatest value of this model is the insertion of quantification idea into matter models, even in an unjustified way**.

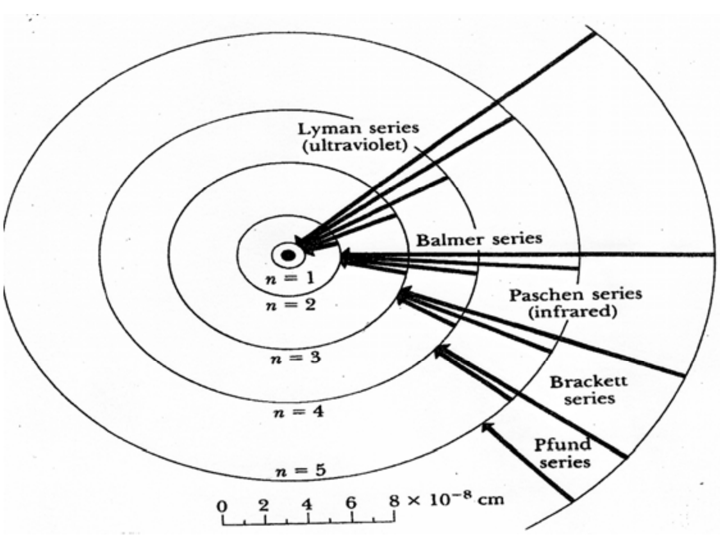
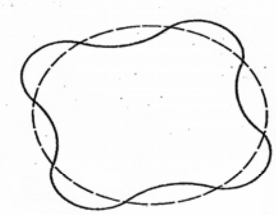


Fig 1



A standing wave set up around the perimeter of a circle. This picture was used by de Broglie to explain the quantization of angular momentum in Bohr's theory.

Fig (2)

- The next step toward the quantum theory is related with the introduction of dual nature into the matter particles. Louis de Broglie did this in 1924. He considered that **the dual nature is a general feature of all particles in sub-atomic world**. Furthermore, he combined the quantum and special relativity ideas for those particles in the same way Einstein did for the photons. So, he proposed **to associate to a particle with linear momentum *p = mv* one wave with wavelength λ such that**

$$\lambda = h / p \qquad (3)$$

Note that (3) is the same as (2) but read in a reverse order.

- The first validity proof of this hypothesis was the justification of “allowed orbits” in Bohr’s model. One of the postulates Bohr did was that the angular momentum of electrons in the allowed orbit “n” must fulfill the condition

$$mvr = n\frac{h}{2\pi}; n = 1,2,3... \qquad (4)$$

Note that Bohr put this condition so that to get one mathematical expression for energy levels which would explain the recorded hydrogen spectra. He did not give any physical reasoning. When de Broglie applied his equation (3) into (4) he got the expression

$$\frac{h}{\lambda} r = n\frac{h}{2\pi}; \rightarrow\rightarrow\rightarrow\rightarrow 2\pi * r = n\lambda; \_ n = 1,2,3... \qquad (5)$$

The condition (5) is *similar to requirement for a standing wave*. So, de Broglie found a interpretation for the arbitrary postulate of Bohr; “ ***Only those orbits that can fit an integral number of e^- wavelength around the circumference are allowed*** (fig 2)”. This explanation is based on the assumption of a wave associated to the electron. As **e-** is a ***matter particle*** the line spectra are an **indirect *proof*** for the existence of **particle waves**.

ELECTRON DIFFRACTION

- The explanation of “allowed orbits” in Bohr’s model was only an indirect proof of matter’s waves. Meanwhile, the direct proof needed a diffraction of interference pattern produced by those waves. Davisson and Germer realised the experimental proof in 1926. In this experiment, a heated filament produced electrons that were provided a linear momentum by acceleration through a potential difference V (fig 3). The **e-** beam was directed on a *Ni target* and the number of reflected electrons along angles was measured.

-Depending on **e-** nature two results may appear:

a) **If** electrons are **particles**, after collision with regularly arranged atoms in the crystalline array of Ni, they **scatter uniformly in space**.

b) **If** electrons are **waves**, after collision with regularly arranged atoms in the crystalline array of Ni, they scatter in space **obeying to diffraction rules**. In this case one must find some **grater number of e- scattered along some several space directions**. But, we know that the wave **diffraction** is produced only **if wavelength is smaller that the slits width** and inters lit distance. Davison & Germer could control the e- related wavelength by use of potential V.

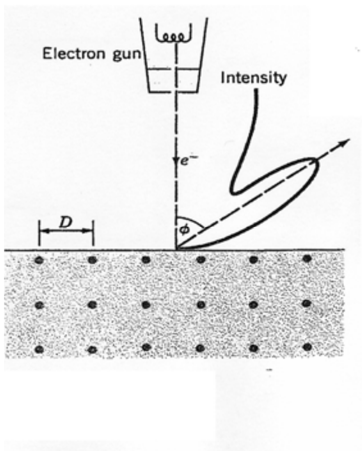


Fig 3

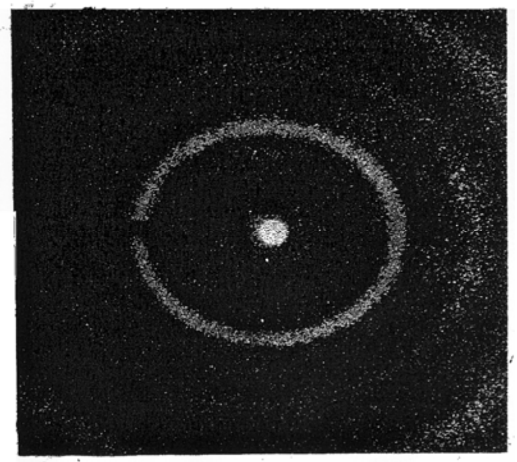


Fig 4

- Note that after acceleration inside the potential V, the **e-** posses the kinetic energy

$$K = \frac{mv^2}{2} = \frac{m^2v^2}{2m} = \frac{p^2}{2m} = eV \_\_ So \_\_ p = \sqrt{2meV} \qquad (6)$$

Based on the expression (3) the corresponding wavelength of e- wave would be

$$\lambda = h / p = h / \sqrt{2meV} \qquad (7)$$

So, one may decrease the wavelength by increasing the value of V. Taking into account that in case of Ni the “inter-slit distance “ is D = 0.215nm they increase V to get smaller values for λ. When Davison & Germer realised that condition they found that the number of reflected electrons was much bigger along several directions of space. **This was a direct proof that electrons posses a wave facet, i.e. a dual nature**. Other experiments proved the **dual particle-wave** nature of **neutrons**(fig 4) and **protons**.

SHREDINGER’S WAVE EQUATION

- At the end of the second decade of 20th century, the **experimental** results had **proved** the ***dual nature for light and subatomic particles***. Also, there were ***theoretic models*** *for the* ***two facets of light*** and only ***for particle behaviour of matter***.
There was no model for the wave behaviour of matter.

- Erwin Shredinger filled this gap. He observed that, *independently on the very different* ***physical meaning*** *of their "****displacement****", the mechanic and light phenomena obey to the* ***same wave equation***. So, at first, he assumed that the **matter waves have to obey to the same propagation equation** (8) independently on their specific meaning.

$$\frac{\partial y^2}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad (8)$$

"y" is the "displacement of matter wave; "v" is its propagation speed
"x" is the space variable; "t" is the time variable

- At first, he considered de Broglie hypothesis for **the stability** of "allowed orbits" in Bohr's model. Noting that the ***electron's wave behaves as a standing wave***, Shredinger remembered that the standing wave function " $y = 2A\sin(\mathrm{kx}) * \cos(\omega t)$ "is expressed as the product of two different functions; the first *space-dependent* and the second *time-dependent*. So, he assumed that the wave function of the electron must have the form

$$y(x,t) = \psi(x) * \cos(\omega t) \qquad (9)$$

The partial derivatives of **y** are: $\frac{\partial y}{\partial x} = \frac{d\psi(x)}{dx}\cos(\omega t); \_ \frac{\partial^2 y}{\partial x^2} = \frac{d^2\psi(x)}{dx^2}\cos(\omega t)$ (10)

and $\frac{\partial y}{\partial t} = -\omega\sin(\omega t)\psi(x); \frac{\partial^2 y}{\partial t^2} = -\omega^2\cos(\omega t)\psi(x)$ (11)

By substituting them into the equation (8) we find $\frac{d^2\psi(x)}{dx^2} = -\frac{\omega^2}{v^2}\psi(x)$ (12)

As $E = K + U = mv^2/2 + U = m^2v^2/2m + U = p^2/2m + U \rightarrow p = \sqrt{2m(E - U}$ (13)

we get $\left(\frac{\omega}{v}\right)^2 = \left(\frac{2\pi}{\lambda}\right)^2$ and $p = \frac{h}{\lambda}$. Then $\left(\frac{\omega}{v}\right)^2 = \left(\frac{2\pi}{h}p\right)^2 = \frac{p^2}{\hbar} = \frac{2m(E-U)}{\hbar}$ (14)

So, the equation (12) becomes $\frac{d^2\psi(x)}{dx^2} = -\frac{2m(E-U)}{\hbar}\psi(x)$ (15)

This is known as ***one-dimensional time-independent Schrödinger wave equation***. Then, he showed that in case of **H atom**, each solution ψ_n of equation (15) corresponds to one of the energy levels E_n given by Bohr's model. This way, this **equation** became the **decisive step** toward the theory of **quantum mechanics**, a theory ***about wave behaviour of matter particles***. Nowadays quantum mechanics is a complete theory.

- It important to note that the physicists spent some time to get the precise **meaning of wave function**. It was Max Born who suggested the interpretation of wave function that is now officially accepted. ***At first***, he underlined that the ***intensity*** of light wave ***at a given point*** of space is a measure of ***number of photons at his point***. As the ***intensity*** is ***proportional*** to the ***square*** of **wave function**, it comes out that the **light wave**

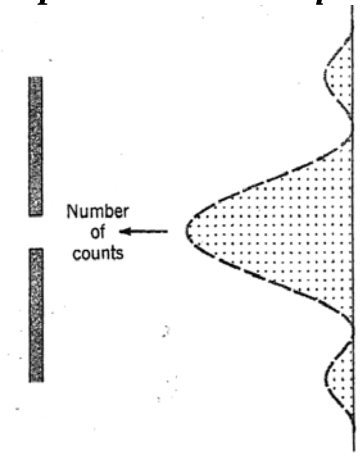


Fig 5

function determines the **probability** to find a photon at any point of space. ***Then***, he noted that the **diffraction** of ***electron wave*** by a narrow slit (fig 5) would produce the ***same patterns as that of photon*** (light) diffraction. The, b***y analogy***, it came out that the intensity of wave function, ψ^2 , determines the **probability** of finding the **e-** (particle) at a given point of space.
His ***last step*** was the determination of the *probability to find the particle within a infinitely small volume* dV as

$$\psi^2 dV \tag{16}$$

and the probability to find the particle some where within the space as

$$\int_{-\infty}^{+\infty} \psi^2(x)dx = 1 \tag{17}$$

- The expression (17) is known as the ***normalization condition*** and ψ^2 (x) as the **density of probability** while the **wave function** presents a ***wave of probability***. Note that the language of quantum mechanics is based on probability. So, there is no sense to talk about the exact position where a particle will be detected.

Mecanic quantum type of comment; It is more likely to observe the particle wherever ψ^2 (x) is large and less likely to observe it wherever ψ^2 (x) is small.

- The quantum **wave equation** (15) concerns only one space coordinate and its solutions are wave functions of one coordinate. When studying a problem in the real space, i.e. three coordinates space, one must work with **three-*dimensional wave equation*s.**
In this case the solutions are functions of three space variables ψ(x,y,z). The solutions of three-dimensional problem for the hydrogen atom are presented in fig 6. Note that these **probability "clouds**":

a) include the circular Bohr orbits;
b) have the same number of nodes as Broglie waves;
c) tell only for possible positions of electrons and not for precise orbits

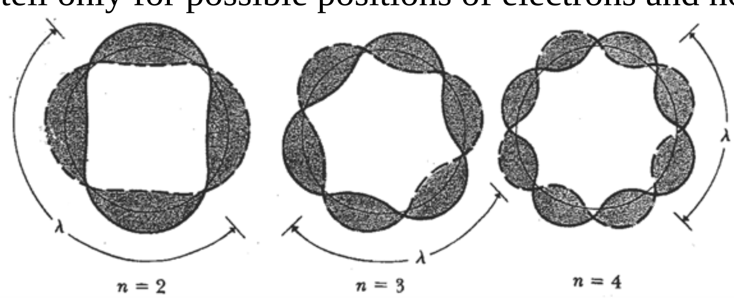


Fig 6

Wrapping of standing waves around Bohr's orbits. So that a standing wave fits to a given circular Bohr's orbit, the perimeter of this orbit must be equal to an integral number of wavelength. Count the number of wavelength for orbits with quantum number n=2,3,4.

THE HEISENBERG UNCERTAINITY PRINCIPLE

-We learnt that light and matter have dual nature and the wave function tells about the probability of finding a particle at a given space position. But, we know that a **particle** is **localized** in space while a **wave is not**. How to adjust this discrepancy?

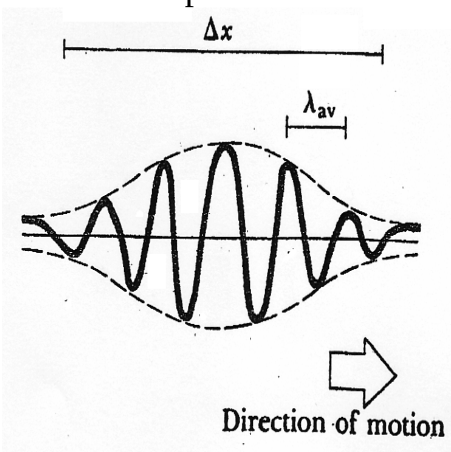


Fig 7

Figure 7 shows what is called a **wave packet**[1] that *has both wave and particle properties*. The regular spacing λ_{av} between successive maxima is a characteristic of wave but it has also a particle like localization order Δx in is space. To understand what way the wave packet is built one has to remember the superposition of ***two waves*** with close values of wavelengths λ,λ' that produces the wave beating. A beating's wave profile contains many wave packets. One may show that a wave packet is produced if one increases the number of superposed waves with close λ-values; λ_{av} that appears in fig 7 is the **average** λ-value of those superposed waves.

- The fact that the wave packet contains many wavelengths means that, when using the expression $\lambda = h/p$ for the wave associated to a particle, one must be conscious that this λ-value is determined with an ***uncertainty*** $\Delta\lambda$. But, this uncertainty brings automatically (due to relation $\lambda = h/p$) the existence of an **uncertainty** Δp for the linear impulse p. So, when talking for a quantum particle, we have to deal with <u>*two uncertainties*</u>, one for the particle position Δx (due to wave packet extension in space) and one for the impulse Δp_x (due to the participation of many λ in the wave packet). *The uncertainty principle of Heisenberg states that: The position and impulse uncertainties of a quantum particle obey to condition*

$$\Delta x * \Delta p_x \geq h \tag{18}$$

-This principle means that we <u>*cannot measure simultaneously*</u> both the position and the impulse of <u>*a quantum particle with an arbitrary precision*</u>. Suppose that during an experiment we measure simultaneously the position and impulse of a quantum particle and we found the uncertainties $\Delta x, \Delta p_x$. Then, we improve the precision for position measurement (*decrease* Δx). The principle (18) tells that automatically the precision for impulse results decreases (Δp_x *increases*). This is a ***fundamental restriction imposed by nature*** and there are no experimental ways to skip it.

-Heisenberg came into this principle by analyzing the process of measurement. Here it is one example. Consider the diffraction of electrons by a single slit(fig 8). We know that the **diffraction patter** is the same as that of a light diffraction. So, if one does this experiment one would find a **minimum number of diffracted electrons** along the direction of "first minimum of diffraction for single slit". At **slit's output**, y-coordinate of such an electron is defined with an uncertainty "**a** =**Δy**". So,

$$\sin\theta = \lambda / a \to \sin\theta = \lambda / \Delta y \to \Delta y = \lambda / \sin\theta \qquad (19)$$

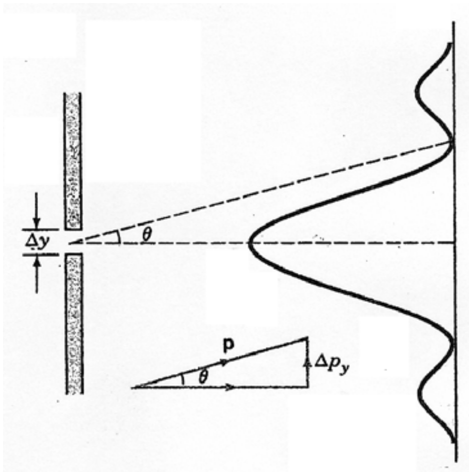


Fig 8

Note that the uncertainty **Δy** of **e-** along Oy direction is equal to the slit's width. The **uncertainty** of linear **momentum** in the Oy direction for all **e-** that fall **inside the central maxima** is (see fig 8) ***psinθ***. But, there are **electrons** that fall out of the central maxima, which have **bigger momentum uncertainty**. So, the **uncertainty** of momentum is **at least** ***psinθ***; mathematically expressed

$$\Delta p_y \geq p\sin\theta \geq \frac{h}{\lambda}\sin\theta \qquad (20)$$

By combining the two expressions (19) and (20) we find that

$$\Delta y * \Delta p_y \geq h \qquad (21)$$

- We considered only the uncertainty principle in relation to position and momentum. But, the principle of uncertainty applies to other couples of variables, too. For example, the uncertainty in time and energy measurement are related similarly

$$\Delta E * \Delta t \geq h \qquad (22)$$

Here are some important comments based on expression (22);

a) One must increase the observation time of one system if one wants more precise measurements of its energy.
b) The **lifetime of an atomic energy level** is a measure of average time that one electron stays at this level before making a transition into a lower energy level. During such a transition, one photon with energy $E = hf = E_{at}^1 - E_{at}^2$ is emitted. Taking into account that $\Delta E = h\Delta f$ and considering Δt the lifetime of upper transition level, one finds out that

$$\Delta E * \Delta t = h\Delta f * \Delta t \geq h \quad \textit{and} \quad \Delta f \geq 1/\Delta t \qquad (22)$$

This result shows that the frequency of emitted photon is sharply defined if the life time of the upper lever is long.