HYPERBOLIC FUNCTIONS

DEFINITION: $\cosh x = \frac{1}{2}(e^{x} + e^{-x})$ EVEN

$\sinh x = \frac{1}{2}(e^{x} - e^{-x})$ ODD

$\tanh x = \frac{\cosh x}{\sinh x}$

$\cosh(x) = \cosh(-x)$, $\sinh(x) = -\sinh(-x)$

- SIMILAR TO TRIG. FUNCTIONS $\cos\theta, \sin\theta, \dots,$ BUT NOT PERIODIC

- RELATION TO TRIGONOMETRIC FUNCTIONS

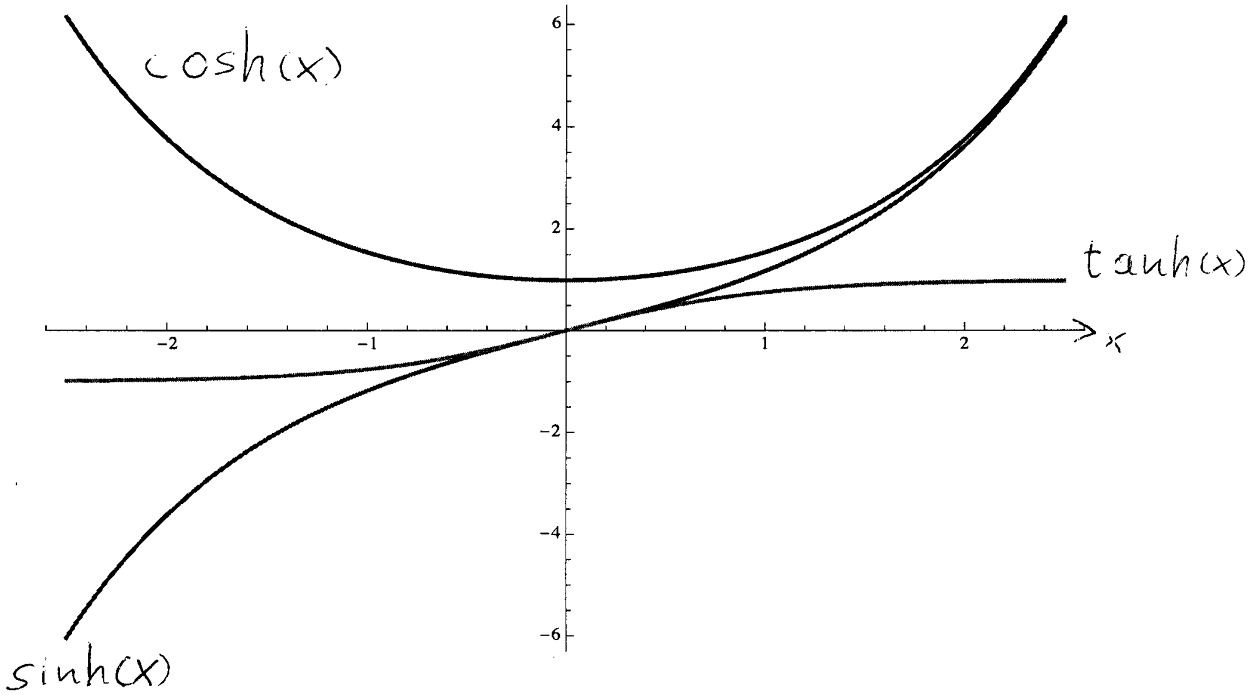
RECALL FROM MT1. $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$

$\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$

NOW SET $\theta = ix$

$$\cos ix = \frac{1}{2}\left(e^{i(ix)} + e^{-i(ix)}\right) = \frac{1}{2}\left(e^{-x} + e^{x}\right) = \cosh x$$

$$\sin ix = \frac{1}{2i}\left(e^{-x} - e^{x}\right) = -\frac{1}{i}\sinh x = i\sinh x$$


cosh(x)
tanh(x)
sinh(x)
x
6
4
2
−2
−4
−6
−2
−1
1
2

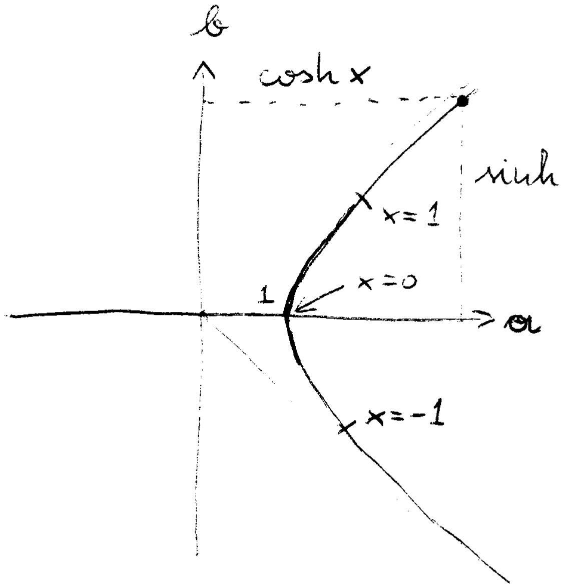
MORE PROPERTIES

- $$\frac{d \cosh x}{dx} = \sinh x \,, \quad \frac{d \sinh x}{dx} = \cosh x$$

- ALSO $\cosh^2 x - \sinh^2 x = 1$
  (PROOF IN HOMEWORK)

  $a = \cosh(x) \,, \quad b = \sinh(x)$

COMPARE WITH
$\cos^2\theta + \sin^2\theta = 1$
EQUATION FOR A UNIT CIRCLE



HYPERBOLA : $a^2 - b^2 = 1$

APPLICATIONS

- SPECIAL RELATIVITY , ELECTRO-MAGNETISM

- CATENARY : SHAPE OF A HANGING CABLE

$$y = \text{const.} + a \cosh\left(\frac{x}{a}\right)$$

- ARCHITECTURE : INVERTED CATENARY ARCH
  # "IDEAL CURVE FOR AN ARCH"