Due April 10, 2008

MAS423/AST001 Solar System
Coursework #4

Potentially useful constants and relations

- Gravitational constant, $G \approx 7 \times 10^{-11}$ N m^2 kg^{-2}
- Specific heat of rock, $c_p \approx 800$ J kg^{-1} K^{-1}
- Melting temperature of rock, $T_{melt} \approx 1000$ K
- Radius of the Earth, $R_\oplus \approx 6,000$ km
- Radius of Ceres, $R_{ceres} \approx 500$ km
- Approximate density of rock, $\rho_{rock} \approx 4000$ kg m^{-3}

1. Because tides raised by satellites in their primaries generally result in the expansion of orbits initially outside the synchronous distance, the presence of a massive satellite in a close orbit set an upper limit on either the age of the satellite or the planet's tidal dissipation function Q. [Repeat from last time]

   (a) Compute the corotation radius for each planet (i.e., the orbital semi-major axis where the mean motion is equal to the rotational angular velocity of the planet). Express this in planetary radii.

   (b) Use the orbital distances and masses of Deimos, Io, Mimas, Ariel, and Proteus to place lower limits on Q of Mars, Jupiter, Saturn and Neptune. Assume the age of the solar system is 4.5×10^9yr, the giant planets are strengthless fluids and Mars has a mean rigidity of $\langle \mu \rangle = 5 \times 10^{-2}$ N m^{-2}.

   (c) Why is the larger satellite Triton not suitable for this calculation for Neptune?

2. This question pertains to energy release and heating during planetary accumulation. [Adapted From dePater and Lissauer]

(a) Show that the total energy released when building a body with mass M, radius R, and bulk density (ρ) by impacting planetesimals is at least:

$$E_o = -\frac{3GM^2}{5R} = -\frac{3GR^5}{5}\left(\frac{4\pi}{3}\rho\right)^2$$

It may be convenient to use the definition of density for a sphere and work in density ρ and r.

(b) Assuming all this energy goes into heat, write an expression for the temperature rise of the planet. Express your answer in terms of density and radius. What is the minimum size a rocky planet must grow to in order to reach the melting temperature by this process? Compare this with the radii of Earth and Ceres and comment on their plausible melting due to this heat source.

(c) Consider that the body differentiates. It continues to have the same bulk density and radius as before, but now has a mantle of density ρ_m overlying a core of radius R_c and density ρ_c. Show that the gravitational potential energy of the differentiated body is

$$E_d = -\frac{3GR^5}{5}\left(\frac{4\pi}{3}\rho_m\right)^2\left(\frac{R_c}{R}\right)^5\left[\left(\frac{\rho_c}{\rho_m}\right)^2 + \left(\frac{R}{R_c}\right)^5 + \frac{5}{2}\left(\frac{\rho_c}{\rho_m} - 1\right)\left(\frac{R^2}{R_c^2} - 1\right) - 1\right]$$

I know it looks ugly. It involves doing just a few simple integrals though, and a little algebra.

(d) Assume that the core is twice as dense as the mantle and that it occupies half the radius of the body. Show that the gravitational energy released by differentiation is

$$E_o - E_d = -\left(\frac{2}{9}\right)^2 E_o.$$

Calculate the temperature rise due to the energy released in the differentiation process. Is the release of energy due to differentiation sufficient to continue melting the Earth in this context?

You should also go through the derivation of the expression for the semi-major axis evolution due to planet tides. You don't need to turn it in, but you should know how to do this derivation.