# LECTURE 21

## Physical applications

**The previous material can be summarized as follows:**

**Gravity is equivalent to curved space-time, hence in all differentials of tensors we should take into account the change in the components of a tensor under an infinitesimal parallel transport. Corresponding corrections are expressed in terms of the Cristoffel symbols and reduced to replacement of any partial derivative by corresponding covariant derivative. In other words we can say that**

**If one wants to take into account all effects of Gravity on any local physical process, described by the corresponding equations written in framework of Special Relativity, one should just replace all partial derivatives by covariant derivatives in these equation according to the following very nice and simple but actually very strong and important formulae:**

$$\mathbf{d} \rightarrow \mathbf{D}$$

**and**

$$, \rightarrow ;$$

Example:

**In special Relativity obviously**

$$dg_{ik} = 0 \text{ and } g_{ik,l} = 0,$$

**while in General Relativity**

$$Dg_{ik} = 0 \text{ and } g_{ik;l} = 0.$$

## The motion of a free particle

**Let us apply above formulae to description of motion of a test particle in a given gravitational field. Let**

$$u^i = \frac{dx^i}{ds}$$

**is the four-velocity. Then equation for motion of a free particle in absence of gravitational field is**

$$\frac{du^i}{ds} = 0$$

**is generalized to the equation**

$$\frac{Du^i}{ds} = 0,$$

**which gives**

$$\frac{Du^i}{ds} = \frac{du^i}{ds} + \Gamma^i_{kn} u^k \frac{dx^n}{ds} = \frac{d^2x^i}{ds^2} + \Gamma^i_{kn} u^k u^n = 0.$$

**Thus from physical point of view the equation**

$$\frac{d^2x^i}{ds^2} + \Gamma^i_{kl} \frac{dx^k}{ds} \frac{dx^l}{ds} = 0$$

**describes the motion of free particle in a given gravitational field and**

$$\frac{d^2x^i}{ds^2} = -\Gamma^i_{kl} \frac{dx^k}{ds} \frac{dx^l}{ds}$$

**is the four-acceleration, while from geometrical point of view this equation is the equation for geodesics in a curved space-time. That is why all particles move with the same acceleration and now this experimental fact is not coincidence anymore but consequence of geometrical interpretation of gravity.**